Math 346 Quiz 1
February 1, 2016

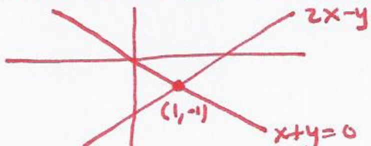
Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided!

1. Solve the system of equations:

   $x + y = 0$
   $2x - y = 3$ $\quad \Rightarrow x =$ 1 , $y =$ -1

2. What is the geometric interpretation of the solution to the above system?
   The solution is the intersection point of the lines x+y=0 and 2x-y=3



3. Define a "function" from a set A to a set B: a rule that assigns to each element in A exactly one element in B

4. Define the "range" of the function above: the set of assigned elements in B.
   OR, the subset of B that is the set of outputs of the function.

5. State the largest possible subset of the real numbers that may be chosen as the domain of the following functions. Also state the corresponding range.

   (a) $f_1(x) = \sqrt{x}$ domain: $[0, \infty)$ range: $[0, \infty)$

   (b) $f_2(x) = \ln x$ domain: $(0, \infty)$ range: $(-\infty, \infty)$

   (c) $f_3(x) = \frac{1}{x}$ domain: $(-\infty, 0) \cup (0, \infty)$ range: $(-\infty, 0) \cup (0, \infty)$

   (d) $f_4(x) = 3x^2 - 2x + 1$ domain: $(-\infty, \infty)$ range: $[2/3, \infty)$

   (e) $f_5(x) = 17 - 5x^3$ domain: $(-\infty, \infty)$ range: $(-\infty, \infty)$

6. (a) Which of the above functions are one-to-one (injective): f_1, f_2, f_3, f_5

   (b) Which of the above functions are onto (surjective): f_2, f_5

   (c) What can we say about a function that is one-to-one and onto? It has an inverse function

7. Let $\mathbb{P}_n$ be the set of all polynomials with real number coefficients of degree n or less.
   Consider the *function* (really "transformation") $f: \mathbb{P}_3 \rightarrow \mathbb{P}_3$ defined by $f(p(x)) = \frac{d}{dx}p(x)$ where $p(x) \in \mathbb{P}_3$.

   (a) Is f one-to-one? NO (b) Is f onto? NO

8. Suppose in problem 7, we had $f: \mathbb{P}_3 \rightarrow \mathbb{P}_2$.

   (a) Is f one-to-one? No (b) Is f onto? Yes

9. Justify your answers to problem 8.

   (a) f is NOT 1-1, since, for example,
   $f(0) = f(1) = 0$
   (i.e. $0 \in \mathbb{P}_2$ does not have a unique preimage)

   (b) f is onto. Since if $P_1(x) = ax^2 + bx + c \in \mathbb{P}_2$ then $f(P_2) = P_1$ where
   $P_2(x) = \frac{a}{3}x^3 + \frac{b}{2}x^2 + cx$.

10. **Bonus Problem:** State as best as you can, Cramer's Rule for a two by two system.

Acceptable answer:

Consider the system of equations

$$ax + by = e$$
$$cx + dy = f$$

Define $D = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, $D_x = \begin{vmatrix} e & b \\ f & d \end{vmatrix}$, $D_y = \begin{vmatrix} a & e \\ c & f \end{vmatrix}$

Then: if $D \neq 0$, the system has a <u>unique</u> solution given by $x = \frac{D_x}{D}$, $y = \frac{D_y}{D}$.

Otherwise, if $D = 0$, the system has either
(I) Infinitely many solutions (if $D_x = D_y = 0$), or
(II) No solution (if $D_x \neq 0$ or $D_y \neq 0$).