Math 346 Quiz 7B

March 21, 2016

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. Let V be a vector space over a scalar field $\mathcal{F}$. Let $\vec{u}, \vec{v} \in V$ and suppose $k, l \in \mathcal{F}$. Which of the following are not a property of V? Circle the appropriate one(s).
   - $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
   - $(k + l)\vec{u} = k\vec{u} + l\vec{u}$
   - $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$
   - $1\vec{u} = \vec{u}$
   - For each $\vec{u} \in V$, if $\vec{u} \neq \vec{0}$, then there exists a vector $\vec{u}^{-1}$ such that $\vec{u} \cdot \vec{u}^{-1} = 1$. (Remember, 1 is the multiplicative identity.)
   - Every $\vec{u}$ has a negative.

2. Determine whether or not the following sets with their operations are vector spaces. If they are, state "yes", if they are not, state what axiom fails.
   (a) The set X of all vectors of the form $< x, y > \in \mathbb{R}^2$, $y > 0$, with the standard operations: No. ① $\vec{0} \notin X$, ② No negatives if $y > 0$. ③ Not closed under scalar multiplication

   (b) The set Y of all vectors $< 0, y > \in \mathbb{R}^2$, with the standard operations: Yes

3. Let $U \subseteq V$, where V is a vector space over $\mathcal{F}$. What two conditions must U fulfill to be a subspace of V?

   (i) If $\vec{u}, \vec{v} \in U$, then $\vec{u} + \vec{v} \in U$. (Closed under addition).

   (ii) If $\vec{u} \in U$ and $k \in \mathcal{F}$, then $k\vec{u} \in U$ (Closed under scalar multiplication).

4. Let $A\vec{x} = \vec{0}$ be a homogeneous system, where $\vec{x}$, $\vec{0} \in V$, where V is a vector space. Let $W \subseteq V$ be the solution set of the system. Prove that W is a subspace of V.

① Closure under addition: If $\vec{x}_1, \vec{x}_2 \in W$, then
$$A(\vec{x}_1 + \vec{x}_2) = A\vec{x}_1 + A\vec{x}_2 = \vec{0} + \vec{0} = \vec{0}$$
$\Rightarrow \vec{x}_1 + \vec{x}_2 \in W$.

② Closure under scalar multiplication: If $\vec{x}_1 \in W$, then
$$A(k\vec{x}_1) = kA\vec{x} = k\vec{0} = \vec{0}$$
$\Rightarrow k\vec{x}_1 \in W$.

$\Rightarrow W$ is a subspace of V. ∎

Bonus:

1. Let $W = \{w_1, w_2, \dots, w_n\}$ be a set of vectors in a vector space V.
   (a) Define what it means to say "W is a basis for V" W spans V and the vectors in W are linearly independent.

   (b) Define what it means to say "W spans V" For every $\vec{v} \in V$, we can write $\vec{v}$ as a linear combination of vectors in W. i.e. there exists $c_1, \dots, c_n$ such that $\vec{v} = c_1\vec{w}_1 + \dots + c_n w_n$

   (c) If W is a basis for V, can you determine the dimension of V? If no, say so. If yes, what's the dimension of V? n