Math 346 Quiz 7A
March 21, 2016

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. Let V be a vector space over a scalar field $\mathcal{F}$. Let $\vec{u}, \vec{v} \in V$ and suppose $k, l \in \mathcal{F}$. Which of the following are not a property of V? Circle the appropriate one(s).
   - $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
   - $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$
   - $(k + l)\vec{u} = k\vec{u} + l\vec{u}$
   - $1\vec{u} = \vec{u}$
   - Every $\vec{u}$ has a negative.
   - For each $\vec{u} \in V$, if $\vec{u} \neq \vec{0}$, then there exists a vector $\vec{u}^{-1}$ such that $\vec{u} \cdot \vec{u}^{-1} = 1$. (Remember, 1 is the multiplicative identity.)

2. Determine whether or not the following sets with their operations are vector spaces. If they are, state "yes", if they are not, state what axiom fails.
   (a) The set X of all vectors of the form $< x, 0 > \in \mathbb{R}^2$ with the standard operations: Yes

   (b) The set Y of all vectors $< x, y > \in \mathbb{R}^2$, $x \geq 0$, with the standard operations: No. ① No negatives if $x > 0$. ② If $\vec{u} \in Y$, $k\vec{u} \notin Y$ if $k < 0$.

3. Let $U \subseteq V$, where V is a vector space over $\mathcal{F}$. What two conditions must U fulfill to be a subspace of V?

   (i) If $\vec{u}, \vec{v} \in U$, then $\vec{u} + \vec{v} \in U$. (Closed under addition).

   (ii) If $\vec{u} \in U$ and $k \in \mathcal{F}$, then $k\vec{u} \in U$ (closed under scalar multiplication)

4. Let $A\vec{x} = \vec{0}$ be a homogeneous system, where $\vec{x}$, $\vec{0} \in V$, where V is a vector space. Let $W \subseteq V$ be the solution set of the system. Prove that W is a subspace of V.

① Closure under addition: If $\vec{x}_1, \vec{x}_2 \in W$, then
$$A(\vec{x}_1 + \vec{x}_2) = A\vec{x}_1 + A\vec{x}_2 = \vec{0} + \vec{0} = \vec{0}$$
$\Rightarrow \vec{x}_1 + \vec{x}_2 \in W$.

② Closure under scalar multiplication: If $\vec{x}_1 \in W$, then
$$A(k\vec{x}_1) = kA\vec{x}_1 = k\vec{0} = \vec{0}.$$
$\Rightarrow k\vec{x}_1 \in W$.

$\Rightarrow W$ is a subspace of V. ∎

Bonus:

1. Let $W = \{w_1, w_2, \dots, w_r\}$ be a set of vectors in a vector space V.
   (a) Define what it means to say "W spans V" For every vector in V, say $\vec{v}$, we can write it as a linear combination of vectors in W. i.e. There exists $c_1, \dots, c_r$ such that $\vec{v} = c_1\vec{w}_1 + \dots + c_r\vec{w}_r$.

   (b) Define what it means to say "W is a basis for V" W spans V and the vectors in W are linearly independent.

   (c) If W is a basis for V, can you determine the dimension of V? If no, say so. If yes, what's the dimension of V? r