

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. Find the following determinants. Direct computation might not be the most efficient way for some of them.

(a) $\begin{vmatrix} 1 & 2 & 1 \\ 1 & -1 & 0 \\ 2 & -2 & 3 \end{vmatrix} = \underline{-9}$

Expansion along second row

(b) $\begin{vmatrix} -2 & -4 & -2 \\ 1 & -1 & 0 \\ 2 & -2 & 3 \end{vmatrix} = \underline{18}$

This is -2 times the answer to (a) since the first row of (a) was multiplied by -2

(c) $\begin{vmatrix} -1 & 1 & -1 \\ -1 & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix} = \underline{-4}$

Used pivotal condensation

(d) $\begin{vmatrix} 7 & 0 & -1 & 1 \\ 2 & 0 & 3 & 0 \\ -3 & 0 & 1 & 2 \\ 1 & 0 & 4 & -1 \end{vmatrix} = \underline{0}$

Has a column of zeros

(e) $\begin{vmatrix} 4 & -3 & 2 & 1 \\ -1 & 0 & 2 & 2 \\ 12 & -9 & 6 & 3 \\ 4 & 7 & -1 & 1 \end{vmatrix} = \underline{0}$

Third row is 3 times the 1st row.

2. A system is augmented as $\left(\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 1 & -1 & 0 & 7 \\ 2 & -2 & 3 & 13 \end{array}\right)$. How many solutions are there? Only one

$\text{Det} \neq 0 \Rightarrow \text{RREF} = I_3$

Bonus:

1. Suppose $A_{2 \times 2}$ is a matrix with $|A| = -3$. What is $\det 2A^3 A^T A^{-1} = \underline{2^2 (-3)^3 (-3) \left(\frac{1}{-3}\right) = 108}$

2. What are the reduced row echelon forms of the matrices in:

(a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$