

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. Solve the following systems, write the answer as a linear combination of column vectors. If no solution exists, write "inconsistent".

(a) $x - y + 3z = 0$
 $x + 3z = 0$
 $2x - y + 6z = 1$

$$\left(\begin{array}{ccc|c} 1 & -1 & 3 & 0 \\ 1 & 0 & 3 & 0 \\ 2 & -1 & 6 & 1 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & -1 \end{array} \right) \begin{array}{l} R_1 \\ R_1 - R_2 \\ 2R_1 - R_3 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} R_1 \\ R_2 / -1 \\ R_2 - R_3 \end{array}$$

⚡ Gasp!

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \text{Inconsistent}$$

(b) $x + 3z = 3$
 $x + y + z = 2$
 $2x + 3y - 2z = 1$

$$\left(\begin{array}{ccc|c} 1 & 0 & 3 & 3 \\ 1 & 1 & 1 & 2 \\ 2 & 3 & -2 & 1 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 3 & 3 \\ 0 & -1 & 2 & 1 \\ 0 & -3 & 8 & 5 \end{array} \right) \begin{array}{l} R_1 \\ R_1 - R_2 \\ 2R_1 - R_3 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 3 & 3 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 2 & 2 \end{array} \right) \begin{array}{l} R_1 \\ -1 \cdot R_2 \\ R_3 - 3R_2 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \begin{array}{l} R_1 - 3(R_3/2) \\ R_2 + R_3 \\ R_3/2 \end{array}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -2 - k + t \\ 7 - 2t \\ k \\ t \end{pmatrix}$$

Bonus:

$x_1 + x_2 + x_3 + x_4 = 5$
 $x_1 + x_3 - x_4 = -2$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 1 & 0 & 1 & -1 & -2 \end{array} \right)$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & -1 & -2 \\ 0 & 1 & 0 & 2 & 7 \end{array} \right) \begin{array}{l} R_2 \\ R_1 - R_2 \end{array}$$

$x_3 = k, x_4 = t$

$R_2: x_2 = 7 - 2t$

$R_1: x_1 = -2 - k + t$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -2 \\ 7 \\ 0 \\ 0 \end{pmatrix} + k \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ -2 \\ 0 \\ 1 \end{pmatrix}$$