Math 346 Quiz 4A
February 22, 2016

Name: ANSWERS

Instructions: No calculators! Answer <u>all</u> problems in the space provided! Do your rough work on scrap paper.

1. Solve the following systems, write the answer as a linear combination of column vectors. If no solution exists, write "inconsistent".

(a)
$$\begin{aligned} x \quad\quad + z &= 1 \\ 2x - 3y + 2z &= 2 \\ -x + y + z &= 3 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 2 & -3 & 2 & 2 \\ -1 & 1 & 1 & 3 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 3 & 0 & 0 \\ 0 & 1 & 2 & 4 \end{array}\right) \begin{array}{l} R_1 \\ 2R_1 - R_2 \\ R_1 + R_3 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & -4 \end{array}\right) \begin{array}{l} R_1 \\ R_2/3 \\ R_2/3 - R_3 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right) \begin{array}{l} R_1 - R_3/2 \\ R_2 \\ R_3/2 \end{array}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$$

(b)
$$\begin{aligned} x + y + 2z &= 0 \\ x - 2y + 2z &= 1 \\ 2x \quad + 4z &= 0 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 1 & -2 & 2 & 1 \\ 2 & 0 & 4 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 3 & 0 & -1 \\ 0 & 2 & 0 & 0 \end{array}\right) \begin{array}{l} R_1 \\ R_1 - R_2 \\ 2R_1 - R_3 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 3 & 0 & -1 \\ 0 & 0 & 0 & -2 \end{array}\right) \begin{array}{l} R_1 \\ R_2 \\ 2R_2 - 3R_3 \end{array}$$

↯ Gasp!

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \text{Inconsistent!}$$

Bonus:

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 2 \\ x_1 \quad + x_3 - x_4 &= -1 \end{aligned}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 1 & 0 & 1 & -1 & -1 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & -1 & -1 \\ 0 & 1 & 0 & 2 & 3 \end{array}\right) \begin{array}{l} R_2 \\ R_1 - R_2 \end{array}$$

$x_3 = k$, $x_4 = t$

R_2: $x_2 = 3 - 2t$

R_1: $x_1 = -1 - k + t$

$$\Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -1 - k + t \\ 3 - 2t \\ k \\ t \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 0 \\ 0 \end{pmatrix} + k \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ -2 \\ 0 \\ 1 \end{pmatrix}$$