Math 308 Quiz 9
July 2, 2015

Name: ANSWERS

Instructions: Answer <u>all</u> problems in the space provided. Show all work.

1. Let R be a relation on $\mathbb{Z}$ defined by aRb iff $a^3 = b^3$. Show that R is an equivalence relation. (5 points)

Pf: Reflexive: Clearly $a^3 = a^3 \ \forall a \in \mathbb{Z}$.
Symmetry: Assume $a^3 = b^3$. Then, by symmetry of "=", $b^3 = a^3$.
Transitive: Assume $a^3 = b^3$ and $b^3 = c^3$. Then, by transitivity of "=", $a^3 = c^3$.

$\Rightarrow R$ is an equivalence relation. ∎

2. What are the equivalence classes of the above? (2 points)

There are an infinite number! $[a] = \{a\} \ \forall a \in \mathbb{Z}$.
That is, each $a \in \mathbb{Z}$ is only related to itself.

3. Let R be a relation on $\mathbb{Z}$ defined by aRb iff $a + 2b \equiv 0 \ (mod\ 3)$. Show that R is an equivalence relation. (5 points)

Pf: Reflexive: Since $a + 2a = 3a \equiv 0 \ (\text{mod } 3)$, R is reflexive.
Symmetry: Assume $a + 2b \equiv 0 \ (\text{mod } 3) \Rightarrow a + 2b = 3k$ for $k \in \mathbb{Z}$. Then $a = 3k - 2b \Rightarrow b + 2a = b + 2(3k - 2b) = 3(-b + 2k) \equiv 0 \ (\text{mod } 3)$.
Transitivity: Assume $a + 2b \equiv 0 \ (\text{mod } 3)$ and $b + 2c \equiv 0 \ (\text{mod } 3)$. Then $a + 2b = 3k$ and $b + 2c = 3l$ for $k, l \in \mathbb{Z}$. Adding these equations give $a + 2c = 3(k + l - b) \equiv 0 \ (\text{mod } 3)$.

∎

4. What are the equivalence classes of the above? (Hint: there are three. Caution and patience) (3 points)

$[0] = \{0, \pm 3, \pm 6, \pm 9, \ldots\}$
$[1] = \{\ldots, -5, -2, 1, 4, 7, 10, \ldots\}$
$[2] = \{\ldots, -7, -4, -1, 2, 5, 8, 11, \ldots\}$

Bonus problems: (2 points each)

1. What is $\mathbb{Z}_3 \cap \mathbb{Z}_5 =$ $\emptyset$

2. Let A and B be sets. Define a *function from A to B* in terms of relations.

A function is a relation from A to B in which each element of A appears as the first coordinate of exactly <u>one</u> ordered pair.