Math 308 Quiz 4

June 10, 2015

Name: ANSWERS

Instructions: Answer all problems in the space provided.

1. Write down the negation of the following logical expressions:

   (a) $\forall x, P(x) \vee Q(x)$: $\exists x, \sim P(x) \wedge \sim Q(x)$ (b) $\exists x, P(x) \wedge Q(x)$: $\forall x, \sim P(x) \vee \sim Q(x)$

   (c) $\forall x, Q(x)$: $\exists x, \sim Q(x)$ (d) $\exists x, P(x) \Rightarrow Q(x)$: $\forall x, P(x) \wedge \sim Q(x)$

2. Negate the following statements:
   (a) For every set A, $A \cap \bar{A} = \emptyset$: There exists a set A such that $A \cap \bar{A} \neq \emptyset$

   (b) There exists a set A such that $\bar{A} \subseteq A$: For all sets A, $\bar{A} \not\subseteq A$

3. Prove that if $3n + 7$ is odd for every integer n, then $4n + 17$ is even.
   Pf: Since "$3n+7$ is odd for every integer n" is false (check $n=1$), the claim holds vacuously. ∎

4. Prove that if a and c are odd, then $ab + bc$ is even for every integer b.
   Pf: Assume a and c are odd. We have two cases: (i) b is odd, (ii) b is even.
   (i) If b is odd, then ab and bc are odd $\Rightarrow$ their sum is even.
   (ii) If b is even, then ab and bc are even $\Rightarrow$ their sum is even. ∎

5. Prove: x is odd $\Rightarrow 9x + 5$ is even.
   Pf: Assume x is odd. Then $x = 2k+1$ for $k \in \mathbb{Z}$.
   Then we have $9x+5 = 9(2k+1)+5 = 2(9k+7)$.
   Since $9k+7 \in \mathbb{Z}$, $9x+5$ is even. ∎

Bonus problems:

1. A direct proof of $P \Rightarrow Q$ involves: 1) Assume P is true, 2) Show that Q is true as a consequence. What does a proof by contrapositive of $P \Rightarrow Q$ involve?
   1) Assume $\sim Q$, 2) Show $\sim P$ as a consequence ($P \Rightarrow Q \equiv \sim Q \Rightarrow \sim P$).

2. Prove: Let $x \in \mathbb{Z}$. If $7x + 5$ is odd, then x is even.
   Pf: Assume, by way of the contrapositive, that x is odd. Then $x = 2k+1$ for $k \in \mathbb{Z}$. Then $7x+5 = 7(2k+1)+5 = 2(7k+6)$. Since $7k+6 \in \mathbb{Z}$, we have $7x+5$ is even, and the claim holds by the contrapositive. ∎