Math 308 Quiz 1
June 3, 2015

Name: ANSWERS

Instructions: Answer all problems in the space provided.

1. What is a set? : A collection of objects (unordered).
2. What is the symbol for the null or empty set?: $\emptyset$
3. Define the empty set: $\emptyset$ = the set containing no elements.
4. Define what is means for a set A to be a subset of a set B: Every element in A is also in B. Or, $x \in A \Rightarrow x \in B$.
5. What expression do we write to mean A is a subset of B? $A \subseteq B$
6. What expression do we write to mean A is a proper subset of B? $A \subset B$
7. Fill in the blanks:
   (a) $\mathbb{N} = \{1, 2, 3, \dots\}$ = Natural numbers
   (b) $\mathbb{Z} =$ $\{0, \pm 1, \pm 2, \pm 3, \dots\}$ = Integers
   (c) $\mathbb{Q} =$ $\{\frac{a}{b} \mid a, b \in \mathbb{Z}, b \neq 0\}$ = Rational numbers
   (d) $\mathbb{C}$ = $\{a + ib \mid a, b \in \mathbb{R}, i^2 = -1\}$ = Complex numbers
8. Describe the set in the form $\{x \mid p(x)\}$ where $p(x)$ is a condition on x:
   (a) $A = \{1,2,3,4, \dots, 88\} =$ $\{x \mid x \in \mathbb{N}, x \leq 88\}$ or $\{x \mid x \in \mathbb{Z}, 1 \leq x \leq 88\}$, etc.
   (b) $B = \{-1, 0, 1, 2, 3\} =$ $\{x \mid x \in \mathbb{Z}, -1 \leq x \leq 3\}$
   (c) $C = (0,3] =$ $\{x \mid x \in \mathbb{R}, 0 < x \leq 3\}$
9. What is the cardinality of each of the above sets? (Write as equations using proper notation):
   (a) $|A| = 88$
   (b) $|B| = 5$
   (c) $|C| = \infty$, in particular $\mathbb{C}$.
10. Let $A = \{1,2,a\}$, find:
    (a) $P(A) =$ $\{\emptyset, \{1\}, \{2\}, \{a\}, \{1,2\}, \{1,a\}, \{2,a\}, \{1,2,a\}\}$
    (b) $|A| =$ 3
    (c) $|P(A)| =$ 8
11. Give examples of sets S such that:
    (a) $S \subseteq P(\mathbb{Z})$: S = $\{\{1\}, \{1,2\}\}$ (there are many correct answers).
    ↳ set of sets of integers
    (b) $S \in P(\mathbb{Z})$: S = $\emptyset$ or $\{1\}$ or $\{1,2\}$ (there are many correct answers).
    ↳ sets of integers.

Bonus problems:

1. For nonempty sets A and B, define $A \times B =$ $\{(a,b) \mid a \in A, b \in B\}$