Name: _____ANSWERS_____

Instructions: Use your own scrap paper. Write your answers in the space provided. Simplify your answers!

1. Consider the differential equation $\dfrac{dy}{dt} = y(y^2 - 4)$. Use qualitative methods to analyze its solution by doing the following:

(a) Find: steady states: $y = \underline{-2 \, ; \, 0 \, ; \, 2}$, inflection points: $y = \underline{-\dfrac{2}{\sqrt{3}} \, ; \, \dfrac{2}{\sqrt{3}}}$

(b) (4 points) Sketch the stability graph. Include the stability arrows and smiley faces as shown in class. Label the axes, steady states and inflection points on the graph.
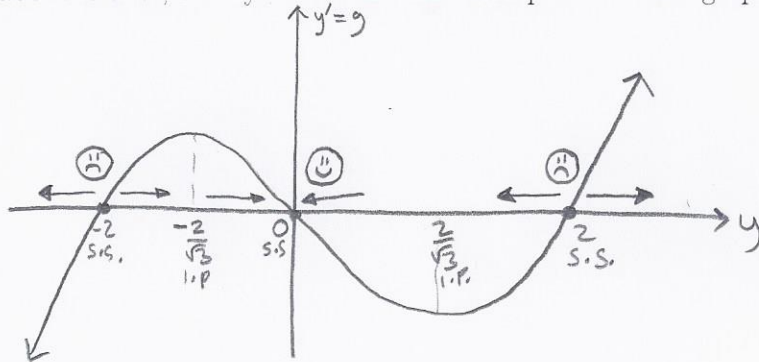
(c) (3 points) Sketch the solution graphs for the solutions having the initial values $y(0) = 1$ and $y(0) = 1.5$. Be sure to label the steady states and inflection points, draw so that concavity is accurately indicated and label all axes.
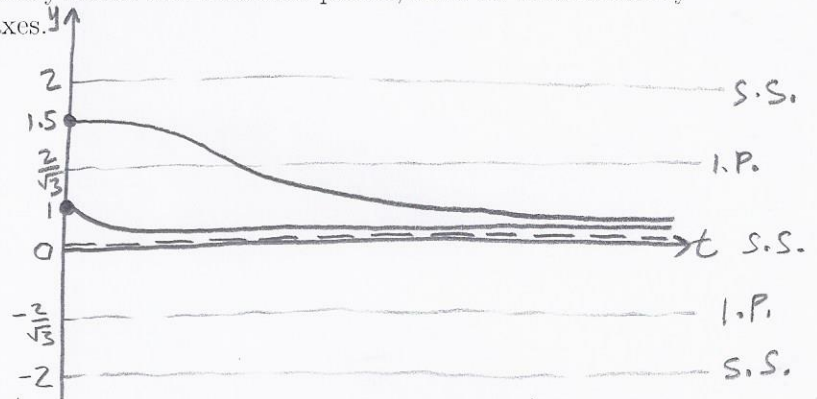
(d) Suppose the given ODE describes a population in units of thousands. What is the long term behavior of this population if you begin with 1.5 thousand individuals? _It will decrease to almost 0 and stay there._

Bonus: State the general ODE formulas for the following:

(a) The Malthus Model: _____$P' = rP$_____

(b) The Harvesting Model: _____$P' = rP - H$_____

(c) The Logistic Model: _____$P' = rP\left(1 - P/k\right)$_____

(d) State the meaning of all variables used: _P = current size of population, r = relative growth rate or growth constant, H = harvesting, k = Carrying Capacity._