MATH 209 TEST 3B

May 6, 2014

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Note that both sides of each sheet have printed material.

Instructions:

1. Read the instructions.
2. Don't panic! Objects in the test are easier than they appear.
3. Complete all problems! Bonus problems will not be counted unless all problems in the actual test are completed.
4. The point values of all problems are indicated.
5. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answer.
6. Where applicable, use 3 decimal places for intermediate steps and write your final answers accurate to two decimal places.
7. Write neatly, so that I am able to follow your sequence of steps. Indicate your answers by boxing them or otherwise.
8. Read through the exam and kill all the easy problems (for you) first!
9. Scientific calculators are needed, but you are NOT allowed to use notes, phones (especially iPhones!), iPads, telepathy, divine inspiration, or other outside aids--including, but not limited to, the smart kid that may be sitting beside you, or the friend you might be thinking of texting.
10. In fact, cell phones should be out of sight. Especially iPhones.
11. Use correct notation and write what you mean! "x^2" and "$x2$" are NOT the same thing, for example. I will grade accordingly.
12. Do NOT commit any of the blasphemies or mistakes I mentioned in the syllabus. I will actually mete out punishment in the way I said I would. I wasn't kidding. From test 1, you guys know I'm not kidding.
13. I am not responsible for you getting anything wrong because you didn't read the above.
14. Other than that, have fun, and good luck! :)

Remember: Save lives by buying more lemons and eating less ice cream!

1. Jhevon poses a challenge to his class: Whoever can stand on his/her head the longest will get an A in the course. Desperate for a good grade, 40 students go into "challenge accepted" mode and attempt the feat. Jhevon, while snickering, measures the time in seconds that students stand on their heads and records the data in the table below.

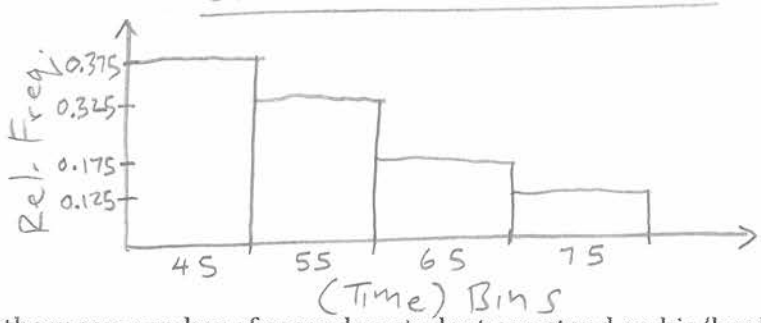
Seconds students can stand on their head	(40,50]	(50,60]	(60,70]	(70,80]
# of students	15	13	7	5
Relative frequency	$\frac{15}{40} = 0.375$	$\frac{13}{40} = 0.325$	$\frac{7}{40} = 0.175$	$\frac{5}{40} = 0.125$
Cumulative frequency	15	28	35	40

2 d.p 0.1<.

(a) Complete the relative frequency row. (2 points)
(b) Complete the cumulative frequency row. (2 points)

(c) Draw a relative frequency histogram for the number of seconds a student can stand on his/her head. (4 points)

Histogram for how long students stand on their heads



(d) Estimate the mean number of seconds a student can stand on his/her head. (4 points)

$$\text{mean} \approx \frac{15(45)+13(55)+7(65)+5(75)}{40}$$

$$= \boxed{55.5 \text{ seconds}}$$

(e) Estimate the median number of seconds a student can stand on his/her head. (8 points)

- median is roughly the 20th #
- This is in the (50,60] bin, 5 #s in
- Step size $= \frac{\text{bin length}}{\text{freq.}} = \frac{10}{13}$

$$\Rightarrow \text{median} \approx (\text{left end point}) + 5(\text{step size})$$

$$= 50 + 5\left(\frac{10}{13}\right)$$

$$\approx \boxed{53.85 \text{ seconds}}$$

2. Consider the four points (2,8), (5,7), (4,4), and (7,5).

(a) Find the following (2 points each):

(i) $\bar{x} = \frac{2+5+4+7}{4} = 4.5$

(ii) $\bar{y} = \frac{8+7+4+5}{4} = 6$

(iii) $s_x = \sqrt{\frac{(2-4.5)^2+(5-4.5)^2+(4-4.5)^2+(7-4.5)^2}{3}} \approx 2.08$ (2.081665999...)

(iv) $s_y = \sqrt{\frac{(8-6)^2+(7-6)^2+(4-6)^2+(5-6)^2}{3}} \approx 1.83$ (1.825741858...)

(v) $c_{xy} = \frac{(2-4.5)(8-6)+(5-4.5)(7-6)+(4-4.5)(4-6)+(7-4.5)(5-6)}{3} = -2$

(vi) $r = \frac{c_{xy}}{s_x s_y} = \frac{-2}{(2.08)(1.83)} \approx -0.53$

(b) Compute the regression line for the given points. (6 points)

$\hat{b} = r \cdot \frac{s_y}{s_x} = -0.53\left(\frac{1.83}{2.08}\right) \approx -0.466$

Using $y - \bar{y} = \hat{b}(x - \bar{x})$

$\Rightarrow y - 6 = -0.466(x - 4.5)$

$\Rightarrow y = -0.466x + 2.097 + 6$

$\Rightarrow y = -0.466x + 8.097$

or

$y = -0.47x + 8.10$

(c) What estimate would you give for the x-value if you know $y = 4$? (2 points)

Using $y = -0.466x + 8.097$

$\Rightarrow 4 = -0.466x + 8.097$

$\Rightarrow x = \frac{4 - 8.097}{-0.466}$

≈ 8.79

OR

Using $y = -0.47x + 8.10$

$4 = -0.47x + 8.1$

$\Rightarrow x = \frac{4 - 8.1}{-0.47}$

≈ 8.72

3. Morpheus places 3 blue pills, 4 red pills, and 1 green pill into a psychedelic urn. Neo must choose two pills **with replacement**. If Neo choses two pills of the same color, then he is very likely "the one." If Neo chooses two blue pills, the story ends and he will wake up in his bed and believe whatever he wants to believe. If Neo chooses two red pills, then he will stay in Wonderland and Morpheus will show him just how deep the rabbit hole goes. If Neo chooses two green pills, Morpheus will throw him off the ship. All Morpheus is offering is the truth—nothing more.

(a) What is the probability that Neo is very likely "the one"? (5 points)

$= P(\text{Same color})$

$= P(\text{2 blue or 2 red or 2 green})$

$= \left(\frac{3}{8} \times \frac{3}{8}\right) + \left(\frac{4}{8} \times \frac{4}{8}\right) + \left(\frac{1}{8} \times \frac{1}{8}\right)$

$= \boxed{\frac{13}{32}}$

(b) What is the probability that Neo is **not** likely "the one"? (5 points)

$= 1 - P(\text{he is likely the one})$

$= 1 - P(\text{same color})$

$= 1 - \frac{13}{32}$

$= \boxed{\frac{19}{32}}$

(c) What is the probability that Neo will stay in Wonderland? (5 points)

$= P(\text{2 red})$

$= \frac{4}{8} \times \frac{4}{8}$

$= \boxed{\frac{1}{4}}$

(d) What is the probability that Neo will **not** get thrown off the ship? (5 points)

$= 1 - P(\text{he gets thrown off})$

$= 1 - P(\text{2 green})$

$= 1 - \left(\frac{1}{8} \times \frac{1}{8}\right)$

$= \boxed{63/64}$

4. Suppose in problem 3, Neo must select pills **without replacement**.

(a) What is the probability that Neo is very likely "the one"? (4 points)

$= P(\text{same color})$

$= P(\text{2 blue or 2 red or 2 green})$

$= \left(\frac{3}{8} \times \frac{2}{7}\right) + \left(\frac{4}{8} \times \frac{3}{7}\right) + 0$

$= \boxed{\frac{9}{28}}$

(b) What is the probability that Neo's story will end and he will wake up and believe what he wants? (4 points)

$= P(\text{2 blue})$

$= \frac{3}{8} \times \frac{2}{7}$

$= \boxed{\frac{3}{28}}$

(c) What is the probability Neo will get thrown off the ship? (4 points)

$= P(\text{2 green})$

$= \boxed{0}$

(d) What is the probability Neo will **not** find out how deep the rabbit hole goes? (4 points)

$= 1 - P(\text{he will find out})$

$= 1 - P(\text{2 red})$

$= 1 - \left(\frac{4}{8} \times \frac{3}{7}\right)$

$= \boxed{11/14}$

(e) What is the probability that Neo will stay in Wonderland? (4 points)

$= P(\text{2 red})$

$= \frac{4}{8} \times \frac{3}{7}$

$= \boxed{\frac{3}{14}}$

5. The statistics are in for a group of students who took either Raymond's Math 205 class or Jhevon's Math 209 class: In Jhevon's class, 10% of students got As. In Raymond's class, 15% of students get As. 8% of students who took both of these classes got As in both. Let J be the event that a student got an A in Jhevon's class. Let R be the event a student got an A in Raymond's class.

(a) Are J and R independent? Explain. (2 points)

No! $P(J \text{ and } R) = 0.08 \neq 0.015 = P(J) \cdot P(R)$

(b) Are J and R mutually exclusive? Explain. (2 points)

No! $P(J \text{ and } R) = 0.8 \neq 0$

(c) What is the probability that a student received an A in Jhevon's class or Raymond's class? (8 points)

This is asking for $P(J \text{ or } R)$

$$P(J \text{ or } R) = P(J) + P(R) - P(J \text{ and } R)$$
$$= 0.10 + 0.15 - 0.08$$
$$= \boxed{0.17}$$

(d) If 20 of the students in the group are randomly selected (perhaps by Neo), what is the probability that at least one got an A in Jhevon's class or Raymond's class? (4 points)

$P(\geq 1 \text{ student got } J \text{ or } R)$
$= 1 - P(0 \text{ students got } J \text{ or } R)$
$= 1 - P(\text{all students did } \underline{\text{not}} \text{ get A in either})$
$= 1 - (1-0.17)^{20} = 1 - (0.83)^{20}$
$\approx \boxed{0.98}$

(e) What is the probability all of the 20 students got As in Jhevon's class or Raymond's class? (4 points)

$P(\text{all 20 had } (J \text{ or } R)) = \boxed{(0.17)^{20}}$

OR $= \boxed{4.06 \times 10^{-16}}$

Bonus 1: Refer to problem 5.

(a) Of the 20 randomly selected students, what is the probability exactly 5 of them got an A in Jhevon's class or Raymond's class? (3 points) This is binomial, $n=20$, $p=0.17$, $q=0.83$.

$\Rightarrow P(X=5) = \binom{20}{5}(0.17)^5(0.83)^{15}$ (X = # that got As in J or R)

$\approx \boxed{0.13}$

(b) Of the 20 randomly selected students, what is the probability of more than 18 of them getting an A in Jhevon's class or Raymond's class? (2 points)

$P(X>18) = P(19) + P(20)$

$= \binom{20}{19}(0.17)^{19}(0.83)^1 + \binom{20}{20}(0.17)^{20}(0.83)^0$

$\approx \boxed{4.01 \times 10^{-14}}$

(c) Of the 20 randomly selected students, what is the probability that at least 2 of them received an A in Jhevon's class or Raymond's class? (3 points)

$P(X \geq 2) = 1 - P(X<2)$

$= 1 - (P(0) + P(1))$

$= 1 - \left[\binom{20}{0}(0.17)^0(0.83)^{20} + \binom{20}{1}(0.17)^1(0.83)^{19}\right]$

$\approx \boxed{0.88}$

Bonus 2: Using the table of Standard Normal probabilities handed out in class, compute the following (1 point each):

(a) $P(Z < 0.6) =$ 0.7257

(b) $P(Z > 1.1) = 1 - 0.8643 =$ 0.1357

(c) $P(-2.1 \leq Z < 1.2) = 0.8849 - 0.0179 =$ 0.867

Bonus 3: Suppose X is a random variable that has a Normal distribution with $\mu = 15$ and $\sigma = 5$.

(a) Write down the formula to find the z-score of a general X-value. (1 point)

$z = \frac{x-\mu}{\sigma}$

(b) What is the z-score for $X = 13$? (2 points)

$z_{13} = \frac{13-15}{5} = -0.4$

(c) What is $P(X > 13)$? (2 points)

$P(X>13) = P(z > -0.4)$

$= 1 - P(z < -0.4)$

$= \boxed{0.6554}$

Typo! $P(X>13)$ not $P(X>17)$, should be obvious though.

Anyway, $P(X>17) = 1 - P(z<0.4)$

$= 0.3446$