

Math 209 Chapter 5 Handout

Population Models

In this chapter we will look at several models used to describe populations. These are:

- The Malthus model: $\frac{dN}{dt} = rN$
- The Harvesting model: $\frac{dN}{dt} = rN - H$ (usually $H > 0$ here, but it is possible to have $H < 0$)
- The Logistic Model: $\frac{dN}{dt} = rN \left(1 - \frac{N}{K}\right)$
where N is the current size of the population, r is the relative growth rate of the population, H is the harvesting of the population (the number of new members being introduced or eliminated per some unit time), and K is the carrying capacity of the population (the optimal size it will tend towards).

We've already seen how to solve equations like the Malthus model and the Harvesting model algebraically via separation of variables. In this chapter we will look at these from a qualitative methods perspective. We also introduce the Logistic model and examine its solution qualitatively. (It is possible to find an exact solution to the logistic model using separation of variables (see problem 12 in HW 8), but we will not use separation of variables for this model in this class. It would involve integration techniques that are not taught in the Math 205-209 track. This means, if a logistic problem shows up on the final or on a test, it will be a qualitative methods problem. You will not be expected nor required to be able to solve this algebraically.)

Examples: Using the above notation, complete the following problems.

1. The Jhevonian population has a birthrate of 31.4 per 1000 per year and a death rate of 2.72 per 1000 per year. Assuming the population grows at a rate proportional to its current size, and it is allowed to grow in peace;
 - (a) Use qualitative methods to analyze the population if it starts with 60221 individuals.
 - (b) Find an equation for the size of the population after t years.
 - (c) What would be the size of the population after 7 years?(Note: This problem requires the use of the _____ model—fill in.)

2. In the seventh year, an evil race of dark Elves wage war with the Jhevonians. Three thousand Jhevonians per year fall in battle. If things continue this way, will the Jhevonians survive? If not, when will they be annihilated?
(Note: This problem requires the use of the _____ model—fill in.)

3. As it turns out, the war never happened. Someone made it up. Punk. Anyway, real world data suggests that the Jhevonian population has a carrying capacity of $K = 200,000$. If the initial population is $N(0) = 60221$, reanalyze the population using qualitative methods.
(Note: This problem requires the use of the _____ model—fill in.)

End of chapter 5. Skip problem 11 in HW 8. Start refreshing your memory on solving systems of equations. It will be important for chapter 6.