MATH 209 TEST 4A

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Note that both sides of each sheet have printed material.

Instructions:

1. Read the instructions.
2. Don't panic! Objects in the test are easier than they appear.
3. Complete all problems! No bonus this time. I spoil you guys too much!
4. The point values of all problems are indicated.
5. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answer.
6. Use 4 decimal places, unless otherwise stated, or it's just silly to do so, like 0.5000.
7. Write neatly, so that I am able to follow your sequence of steps. Indicate your answers by boxing them or otherwise.
8. Read through the exam and kill all the easy problems (for you) first!
9. Scientific calculators are needed, but you are NOT allowed to use notes, phones (especially iPhones!), iPads, telepathy, divine inspiration, or other outside aids--including, but not limited to, the smart kid that may be sitting beside you, or the friend you might be thinking of texting.
10. In fact, cell phones should be out of sight. Especially iPhones.
11. Use correct notation and write what you mean! "x^2" and "$x2$" are NOT the same thing, for example. I will grade accordingly.
12. Do NOT commit any of the blasphemies or mistakes I mentioned in the syllabus. I will actually mete out punishment in the way I said I would. I wasn't kidding. From test 1, you guys know I'm not kidding.
13. I am not responsible for you getting anything wrong because you didn't read the above.
14. Other than that, have fun, and good luck! :)

Remember: Don't take life too seriously. You'll never make it out of it alive!

1. The Honey Badger is just craaazy! It's referred to by the Guinness Book of World Records as the most fearless animal in all the animal kingdom. Overall, there is an 88% chance that *Honey Badger don't care* [and has no regard for any other animal whatsoever]. It just takes what it wants. Assume Honey Badgers are all independently badasses.[1] Suppose you manage to gather 50 Honey Badgers (because you're *that* crazy). Let X = # of Honey Badgers that don't care.

(a) What is the probability that all the Honey Badger don't care? (5points)

$$P(X=50) = (0.88)^{50} = 0.001675 \approx \boxed{0.0017}$$

(b) What is the probability that exactly 42 Honey Badger don't care? (5 points)

$$P(X=42) = \binom{50}{42}(0.88)^{42}(0.12)^{8} \approx \boxed{0.1075}$$

(c) What is the probability that between 30 and 32 (inclusive) Honey Badger don't care? (10 points)

$$P(30 \le X \le 32) = P(30) + P(31) + P(32)$$
$$= \binom{50}{30}(0.88)^{30}(0.12)^{20} + \binom{50}{31}(0.88)^{31}(0.12)^{19} + \binom{50}{32}(0.88)^{32}(0.12)^{18}$$
$$\approx \boxed{1.0277 \times 10^{-5}}$$

(d) What is the **mean <u>and</u> standard deviation** for Honey Badgers that don't care? (Include the formula in your answer. Use two decimal places.)

(2 points) Mean = $np = 50(0.88) = \boxed{44}$

(3 points) Standard deviation = $\sqrt{npq} \approx \boxed{2.2978}$

(e) <u>Estimate</u> the probability that more than 47 Honey Badger don't care. (15 points)

Use the normal approximation with continuity correction.

$$P(X > 47) \approx P\left(z > \frac{46.5 - 44}{2.2978}\right)$$
$$= P(z > 1.09)$$
$$= 1 - 0.8621$$
$$= \boxed{0.1379}$$

[1] Yes, I can say that word. Badass is in the dictionary and everything!

2. Blood types O, A, B and AB occur in the population at frequencies of 0.43, 0.41, 0.11, and 0.05, respectfully. There is also a blood protein called the Rh-factor that occurs in two varieties for all blood types (Rh+ and Rh-) at frequencies of 0.86 and 0.14, respectively. Now a person may receive blood from a donor that has the same Rh-factor and the same blood type, or blood type O.

(a) What is the probability that a random person can donate blood to a person that has blood type A and Rh-? (10 points)

$$P[(A \text{ and } Rh-) \text{ or } (O \text{ and } Rh-)]$$
$$= P(A)\cdot P(Rh-) + P(O)\cdot P(Rh-)$$
$$= 0.41(0.14) + 0.43(0.14)$$
$$= \boxed{0.1176}$$

(b) Suppose a hospital needs 5 units of blood to give to a patient having blood type A and Rh-factor. If 8 people enter the blood donation center and each person can give only 1 unit of blood, what is the probability that,

(i) At least 1 unit of blood can be found for the patient? (10 points) X = # of people who can donate

$$P(X \geq 1) = 1 - P(X=0)$$
$$= 1 - (0.8824)^8$$
$$\approx \boxed{0.6324}$$

(ii) The hospital will be able to obtain all the blood that it needs? (15 points)

$$P(X \geq 5) = P(5) + P(6) + P(7) + P(8)$$
$$= \binom{8}{5}(0.1176)^5(0.8824)^3 + \binom{8}{6}(0.1176)^6(0.8824)^2 + \binom{8}{7}(0.1176)^7(0.8824)^1 + (0.1176)^8$$
$$\approx 9.2531 \times 10^{-4}$$
$$\approx \boxed{0.0009}$$

(iii) The hospital will obtain exactly the amount of blood it needs and no more? (5 points)

$$P(X=5) = \binom{8}{5}(0.1176)^5(0.8824)^3$$
$$\approx 8.6541 \times 10^{-4}$$
$$\approx \boxed{0.0009}$$

3. Suppose X is normally distributed with mean 12 and standard deviation 6.

(a) Compute $P(X \geq 14.5)$ (5 points)

$$z_{14.5} = \frac{14.5 - 12}{6} \approx 0.42$$

$$\Rightarrow P(X \geq 14.5) = P(z \geq 0.42)$$

$$= 1 - 0.6628$$

$$= \boxed{0.3372}$$

(b) What is $P(8 \leq X \leq 13)$? (10 points)

$$z_8 = \frac{8 - 12}{6} \approx -0.67 \qquad z_{13} = \frac{13 - 12}{6} \approx 0.17$$

$$\Rightarrow P(8 \leq X \leq 13) = P(-0.67 \leq z \leq 0.17)$$

$$= 0.5675 - 0.2514$$

$$= \boxed{0.3161}$$

(c) What is the probability that X is less than 3? (5 points)

$$z_3 = \frac{3 - 12}{6} = -1.5$$

$$\Rightarrow P(X < 3) = P(z < -1.5)$$

$$= \boxed{0.0668}$$