MATH 209 TEST 2A

March 18, 2015

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Note that both sides of each sheet have printed material.

Instructions:

1. Read the instructions.

2. Don't panic!

3. Complete all problems! Bonus problems will not be counted unless all problems in the actual test are completed.

4. Note that each problem in the test is worth 20 points. The point values of the bonus problems are indicated.

5. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answer.

6. Write neatly, so that I am able to follow your sequence of steps. Indicate your answers by boxing them or otherwise.

7. Read through the exam and kill all the easy problems (for you) first!

8. Scientific calculators are needed, but you are NOT allowed to use notes, phones (especially iPhones!), iPads, telepathy, divine inspiration, or other outside aids--including, but not limited to, the smart kid that may be sitting beside you, or the friend you might be thinking of texting.

9. In fact, cell phones should be out of sight. Especially iPhones.

10. Use correct notation and write what you mean! "x^2" and "$x2$" are NOT the same thing, for example. I will grade accordingly.

11. Do NOT commit any of the blasphemies or mistakes I mentioned in the syllabus. I will actually mete out punishment in the way I said I would. I wasn't kidding. From test 1, you guys know I'm not kidding.

12. Other than that, have fun, and good luck! :)

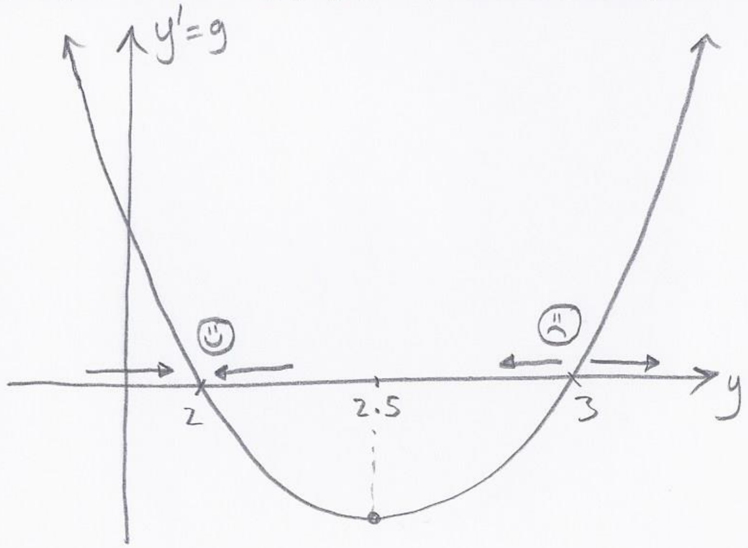
Remember: ...because this is the test this class deserves, and also the one it needs right now.

1. Use geometric analysis to analyze the differential equation $\frac{dy}{dt} = y(y-5)+6$, by:

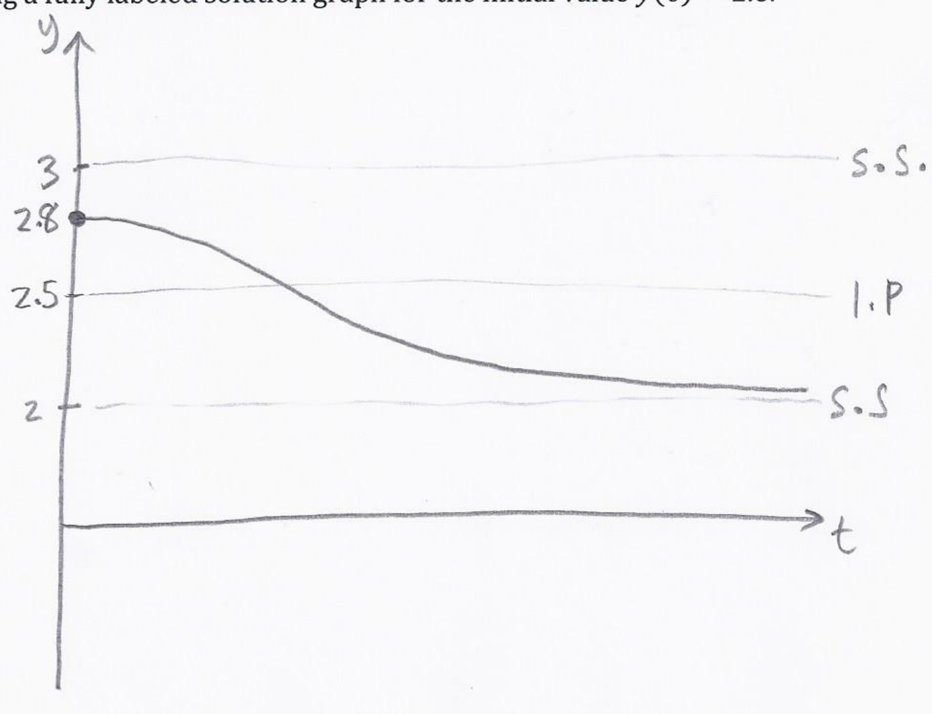
(i) Finding the steady state solutions.

$$g = y(y-5)+6$$
$$= y^2 - 5y + 6$$
$$= (y-3)(y-2)$$
$$\Rightarrow S.S.: \boxed{y=2,\ y=3}$$

(ii) Drawing a fully labeled stability graph, complete with stability arrows.



(iii) Sketching a fully labeled solution graph for the initial value $y(0) = 2.8$.



2. Use geometric analysis to analyze the differential equation $\frac{dy}{dt} = (y+1)(y-2)^2$, by:

(i) Finding the steady state solutions.

$g = (y+1)(y-2)^2$

$\Rightarrow \boxed{y=-1,\ y=2 \text{ are S.S.}}$

For graph: inflection points

$g = (y+1)(y^2-4y+4)$

$= y^3 - 4y^2 + \cancel{4y} + y^2 - \cancel{4y} + 4$

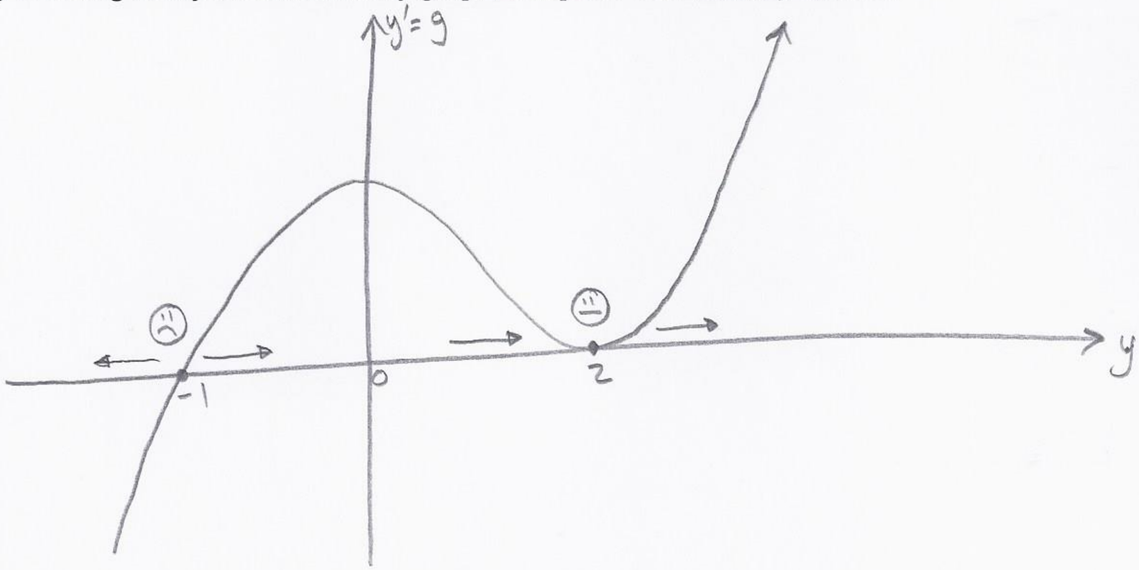
$= y^3 - 3y^2 + 4$

$\Rightarrow g' = 3y^2 - 6y$

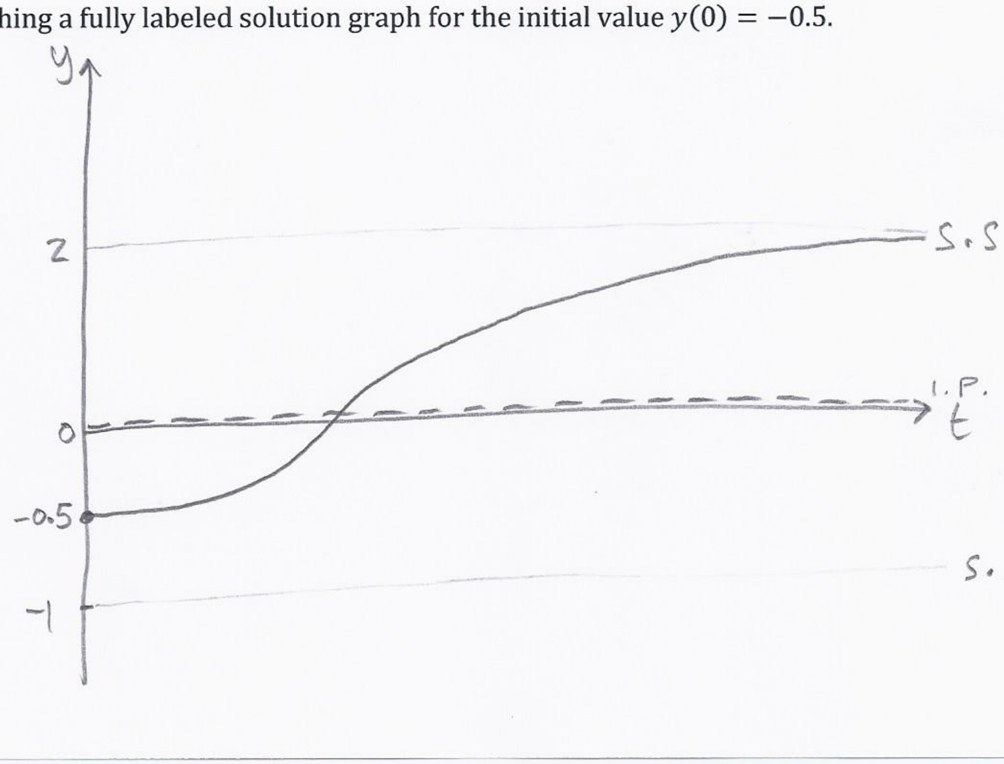
$= 3y(y-2)$

$\Rightarrow y=0,\ y=2 \longrightarrow$ I.P.

(ii) Drawing a fully labeled stability graph, complete with stability arrows.



(iii) Sketching a fully labeled solution graph for the initial value $y(0) = -0.5$.



3. An ecosystem containing two species is modeled by the system of differential equations given below, where N_1 and N_2 denote the number of members of each species and the rates are annual rates of change of the species populations:

$$\frac{dN_1}{dt} = 0.37N_1\left(1 - \frac{N_1}{75} - \frac{N_2}{25}\right) \quad Ⓐ$$
$$\frac{dN_2}{dt} = 0.49N_2\left(1 - \frac{N_2}{50} - \frac{N_1}{25}\right) \quad Ⓑ$$

(a) Find all steady-state solutions of this system.

From Ⓐ: $N_1 = 0$ or $1 - \frac{N_1}{75} - \frac{N_2}{25} = 0$

$\Rightarrow N_1 = 75 - 3N_2$

In Ⓑ: If $N_1 = 0$

$\Rightarrow N_2 = 0$ or $1 - \frac{N_2}{50} = 0$

$\Rightarrow N_2 = 50$

$\Rightarrow \boxed{(0,0),\ (0,50)} \to$ S.S.

If $N_1 = 75 - 3N_2$

$\Rightarrow N_2 = 0$ or $1 - \frac{N_2}{50} - \frac{(75-3N_2)}{25} = 0$

$\Rightarrow 50 - N_2 - 2(75 - 3N_2) = 0$

$\Rightarrow 50 - N_2 - 150 + 6N_2 = 0$

$\Rightarrow -100 + 5N_2 = 0$

$\Rightarrow N_2 = 20$

$\Rightarrow \boxed{(75,0),\ (15,20)} \to$ S.S.

(b) State and justify whether or not the species are competitive.

Yes, it is competitive, by:

Lo's method

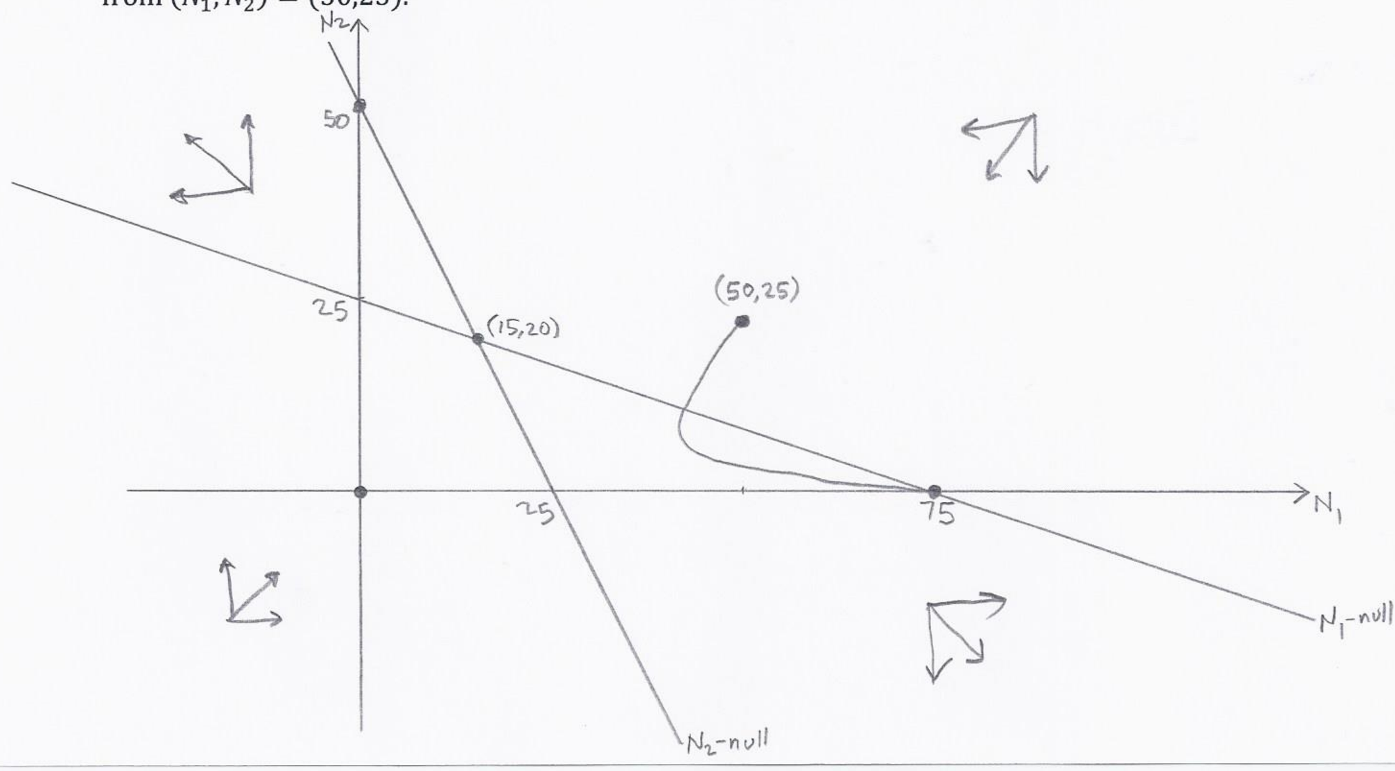
depends

OR

competition table

	1	2
1	$\frac{1}{75} = 0.013$	$< \frac{1}{25} = 0.04$
2	$\frac{1}{25} = 0.04$	$> \frac{1}{50} = 0.02$
Total	0.053	0.06

(c) Draw a fully labeled null-cline diagram for the system, complete with a phase plot starting from $(N_1, N_2) = (50,25)$.



4. Jennifer has a SpongeBob fish farm in her back yard. She has 5000 fish that reproduce at a rate of 5% per year. She harvests 500 fish per year to have with her platanos and salami mega lunch.

(a) Write down a differential equation, with initial condition, to model Jennifer's SpongeBob population.

$$\frac{dN}{dt} = 0.05N - 500 \quad , \quad N(0) = 5000$$

(b) Analyze the population qualitatively by:

(i) Finding the steady states.

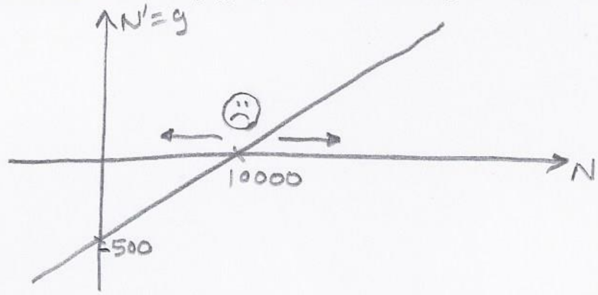
$g = 0.05N - 500$

$= 0.05(N - 10000)$

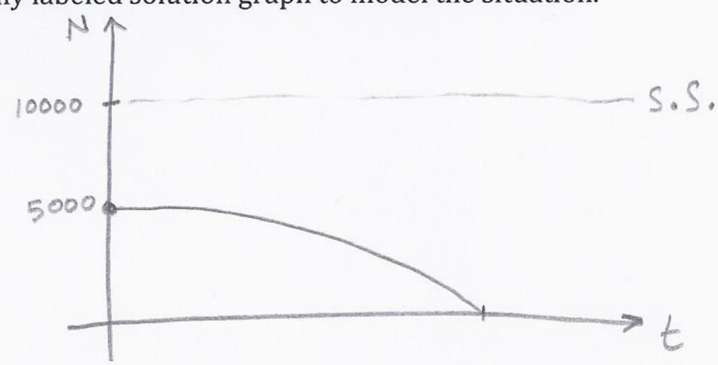
$\Rightarrow \boxed{N = 10000} \rightarrow$ S.S.

(Note: $g' = 0.05 \neq 0$ $\Rightarrow$ no inflections)

(ii) Sketching a fully labeled stability graph with stability arrows.



(iii) Sketching a fully labeled solution graph to model the situation.



(c) Can Jennifer continue to have SpongeBob platanos and salami indefinitely?

No

(d) If not, when will she run out of SpongeBob?

$\frac{dN}{dt} = 0.05(N - 10000)$

$\Rightarrow \int \frac{dN}{N - 10000} = \int 0.05\, dt$

$\Rightarrow \ln|N - 10000| = 0.05t + C$

$\Rightarrow N - 10000 = Ce^{0.05t}$

$\Rightarrow N = Ce^{0.05t} + 10000$

$N(0) = 5000 \Rightarrow C = -5000$

$\Rightarrow N = -5000e^{0.05t} + 10000$

when $N = 0$, we get

$0 = -5000e^{0.05t} + 10000$

$\Rightarrow 2 = e^{0.05t}$

$\Rightarrow t = \frac{\ln 2}{0.05}$

$\approx \boxed{13.86 \text{ years later}}$

5. A population grows logistically at a rate of 4% with a carrying capacity of 600.

(a) Write down an ODE to model this population.

$$\frac{dN}{dt} = 0.04N\left(1 - \frac{N}{600}\right)$$

(b) Find the steady-state solutions of the population.

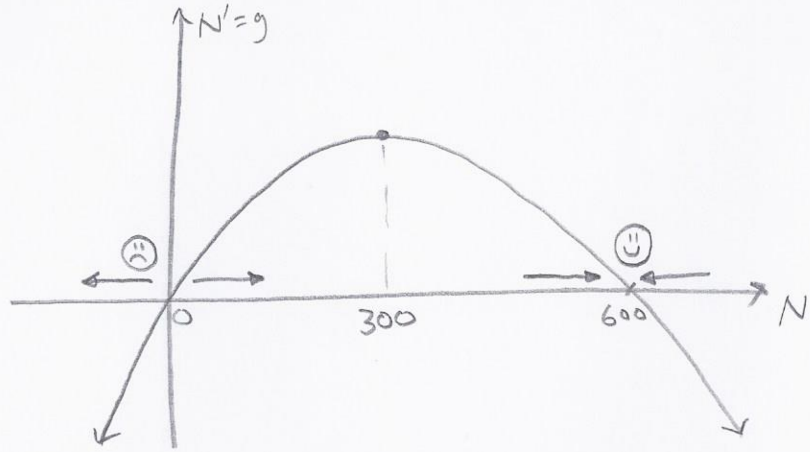
$N=0$ or $1 - \frac{N}{600} = 0$

$\Rightarrow N = 600$

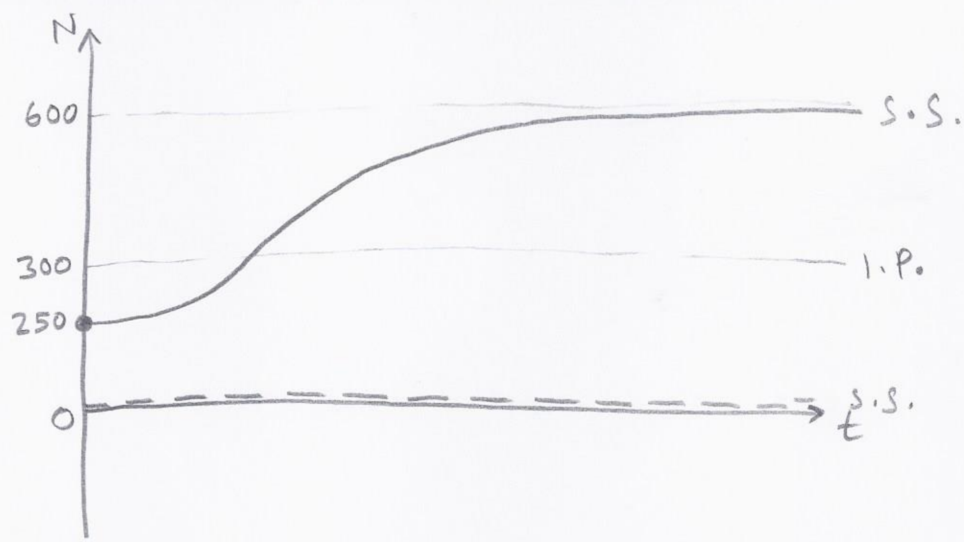
$\Rightarrow$ S.S.: $\boxed{N=0, N=600}$

(c) Perform geometric analysis on the population by:

(i) Drawing a completely labeled stability graph with stability arrows.



(ii) Drawing a fully labeled solution graph, assuming the population is initially 250.



(d) What is the long term behavior of the population?

The population will grow until it reaches the carrying capacity of 600.

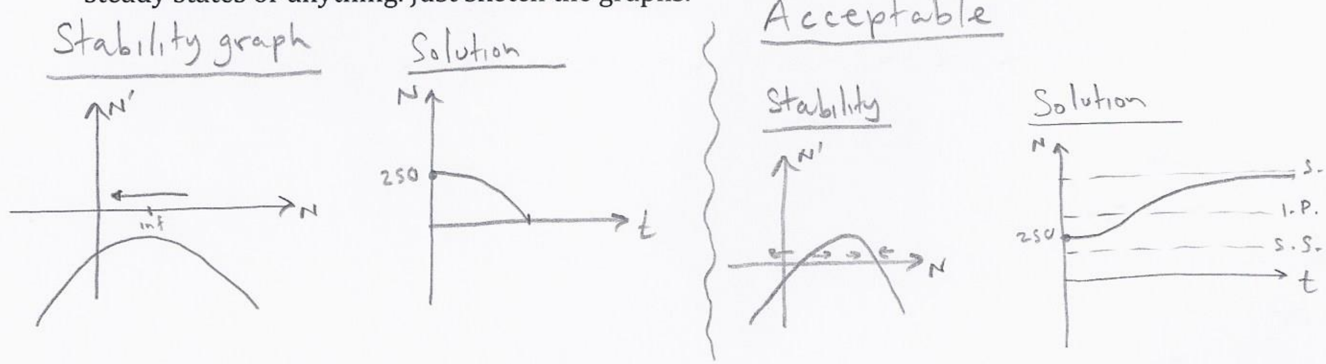
Bonus 1: There is a population model called the "Real World" model. It is simply a logistic model with harvesting.

(a) (2 points) Write down the general form of an ODE for a Real World model.

$$\frac{dN}{dt} = rN\left(1-\frac{N}{K}\right) - H$$

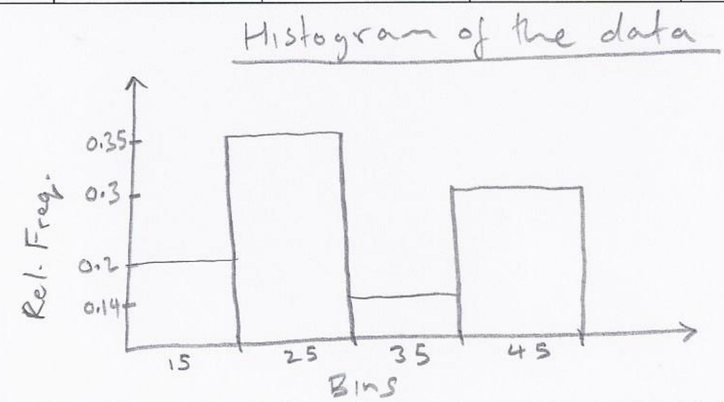
5 better

(b) (8 points) Suppose in problem 5 you had a harvesting of 300 per year. Sketch what the stability and solution graphs would look like (label which is which!). You don't need to find steady states or anything. Just sketch the graphs.



Bonus 2: (5 points) Draw a fully labeled relative frequency histogram for the following data.

Bins	(10, 20]	(20,30]	(30,40]	(40,50]
Frequency	20 0.2	35 0.35	14 0.14	30 0.3



Bonus 3: (1 point each response) For the numbers 7,9,8,7,3,4,3,11,12,14,9,3,2,1; find the:

Mean 6.64, Mode 3, Median 7

Standard deviation 4.07, q1 3, q3 9

Suppose I added 5 to all the numbers in the list. What are the new,

Mean 11.64, Mode 8, Median 12,

Standard deviation 4.07