MATH 209 QUIZ 4 - Version A

March 9, 2015

Name: ANSWERS

Instructions: **Write your answers in the space provided. Do not show calculations on this page.**

1. Write down general formulas for the following models:

(a) The Malthus model: $\frac{dP}{dt} = rP$

(b) The Harvesting model: $\frac{dP}{dt} = rP - H$

(c) The Logistic model: $\frac{dP}{dt} = rP(1 - P/K)$

2. A fish population has initially 30,000 members. The population grows at a rate proportional to its current size and increases by 20% per year. Fishermen fish an average of 10,000 fish per year from this population.

(a) Write down an ODE and initial condition for this population.

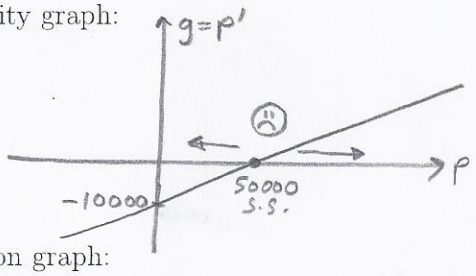
ODE: $\frac{dP}{dt} = 0.2P - 10000$ Initial condition: $P(0) = 30000$

(b) Perform qualitative analysis on this population by:

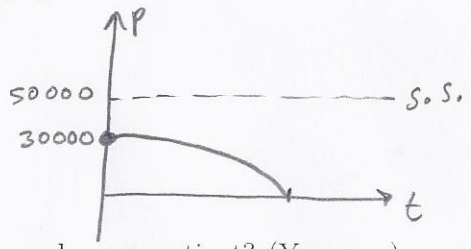
(i) Finding its steady states: $P = 50000$

(ii) Finding its inflection points: None

(iii) Drawing a fully labeled Stability graph:



(iv) Drawing a fully labeled solution graph:



(c) By the above, does the population ever become extinct? (Yes or no) Yes

(d) If it becomes extinct, use quantitative methods to find exactly when (2 decimal places). If not, write "NEVER!" 4.58 years

3. A population N starts with 350 individuals and grows logistically with a growth constant $r = 0.04$ and a carrying capacity of $K = 400$.

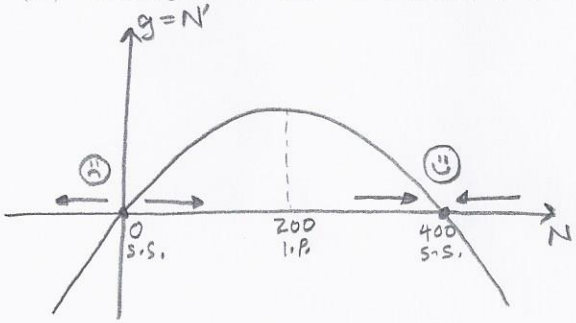
(a) Write down an ODE and initial condition for this population.

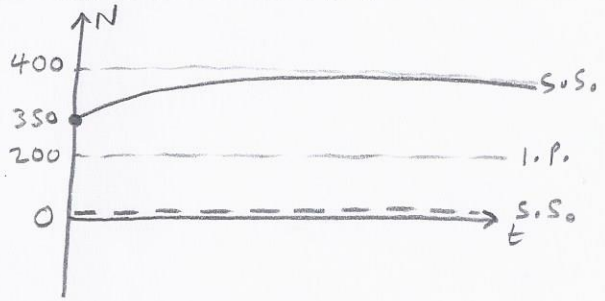
ODE: $\frac{dN}{dt} = 0.04N\left(1 - \frac{N}{400}\right)$ Initial condition: $N(0) = 350$

(b) Perform qualitative analysis on this population by:

(i) Finding its steady states: $N=0$, $N=400$ (ii) Its inflection points: $N=200$

(iii) Drawing a fully labeled Stability graph below. (iv) Draw a fully labeled Solution graph.





(c) Describe the long term behavior of the population: It will increase up to 400, the carrying capacity.

4. Solve the system for x and y:
$$\begin{aligned} x(50 - x - y) &= 0 \\ y(100 - 4y - 5x) &= 0 \end{aligned}$$

$\Longrightarrow (x, y) =$ $(0,0), (0,25), (50,0), (-100,150)$ (List all solutions)

Bonus: (a) Fill out the competition table for:

$$\frac{dN_1}{dt} = 0.23N_1\left(1 - \frac{N_1}{300} - \frac{N_2}{100}\right)$$

$$\frac{dN_2}{dt} = 0.54N_2\left(1 - \frac{N_2}{250} - \frac{N_1}{100}\right)$$

	1	2
1	$\frac{1}{300} = 0.003$	$\frac{1}{100} = 0.01$
2	$\frac{1}{100} = 0.01$	$\frac{1}{250} = 0.004$
Total	0.013	0.014

(b) Is the system above competitive (Yes or No)? Yes