Math 205 Quiz 9
October 25, 2016

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. Let $f(x)$ and $g(x)$ be differentiable functions of x, c a constant. Complete the following formulas. (You may use f' and g' as shorthand):

(a) $\frac{d}{dx}(x^n) =$ nx^{n-1} (b) $\frac{d}{dx}e^u =$ $u'e^u$ (c) $\frac{d}{dx}(a^u) =$ $u'a^u \ln a$

(d) $\frac{d}{dx}\ln u =$ $\frac{u'}{u}$ (e) $\frac{d}{dx}(f(x)\cdot g(x)) =$ $f'g + fg'$

(f) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) =$ $\frac{f'g - fg'}{g^2}$ (g) $\frac{d}{dx}f(g(x)) =$ $f'(g(x))\cdot g'(x)$

2. Differentiate, or find y' in, the following (you don't have to simplify):

(a) $\frac{d}{dx}\frac{\sqrt{x}(x+1)}{x^2e^x} =$ $y\left(\frac{1}{2x} + \frac{1}{x+1} - \frac{2}{x} - 1\right)$ * (b) $\frac{d}{dx}\left[x^x + e^{x^3}\right] =$ $x^x(\ln x + 1) + 3x^2e^{x^3}$

(c) $\frac{d}{dx}\frac{3}{x^2+1} =$ $-6x(x^2+1)^{-2}$ (d) $e^{x+2y} - \ln x = 7 \Rightarrow y' =$ $\frac{\frac{1}{x} - e^{x+2y}}{2e^{x+2y}}$ OR $\frac{1 - xe^{x+2y}}{2xe^{x+2y}}$

(e) $\frac{d}{dx}\left(\ln\left(\frac{e^x}{x}\right)\right)^2 =$ $2\left(\ln\left(\frac{e^x}{x}\right)\right)\cdot\left(1 - \frac{1}{x}\right)$ (f) $\frac{d}{dx}xe^x\ln x =$ $e^x\ln x + xe^x\ln x + e^x$

(g) $xy + \frac{x}{y} + e^y + \ln x = x^2 \Rightarrow y' =$ $\frac{2x - y - \frac{1}{y} - \frac{1}{x}}{x - \frac{x}{y^2} + e^y}$

Bonus:

1. Let $P(t)$ be the amount, in grams, at time t (in years) of a radioactive substance with half-life 32 years. Assume you start with 25 grams of the substance.

(a) Find the differential equation that describes $P(t)$: $P' = -\frac{\ln 2}{32}P$

(b) Find $P(t) =$ $25e^{-\frac{\ln 2}{32}t}$

(c) How much of the substance will remain after 6 months? $25e^{-\frac{\ln 2}{32}\left(\frac{1}{2}\right)} = 25e^{-\frac{\ln 2}{64}}$

* $y = \frac{\sqrt{x}(x+1)}{x^2e^x}$