Math 205 Test 4
December 12, 2016

Name: ___________________________________SOLUTIONS____

Note that both sides of each page may have printed material.

Instructions:

1. Read the instructions.
2. Panic!!! Kidding, don't panic! I repeat, do NOT panic!
3. Complete all problems in the actual test. Bonus problems are, of course, optional. And they will only be counted if all other problems are attempted.
4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers.
5. Write neatly so that I am able to follow your sequence of steps and box your answers.
6. Read through the exam and complete the problems that are easy (for you) first!
7. Scientific calculators NOT allowed and you are NOT allowed to use notes, or other aids—including, but not limited to, divine intervention/inspiration, the internet, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.
8. In fact, **cell phones should be out of sight!**
9. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.
10. Other than that, have fun and good luck!

You survived to the end of Math 205??

1. (10 points each) Evaluate the following integrals. Fully simplify your answers.

(a) $\int \frac{2}{5x\sqrt[3]{\ln x}}\,dx$

$u = \ln x$

$du = \frac{1}{x}dx$

$\Rightarrow dx = x\,du$

$\Rightarrow \int \frac{2}{5\cancel{x}\sqrt[3]{u}} \cancel{x}\,du$

$= \frac{2}{5}\int u^{-1/3}du$

$= \frac{\cancel{2}}{5}\frac{3}{\cancel{2}}u^{2/3} + C$

$= \boxed{\frac{3}{5}(\ln x)^{2/3} + C}$

(b) $\int 3x^{-4}\left(\frac{1}{x^3}+4\right)^5 dx$

$u = x^{-3} + 4$

$du = -3x^{-4}dx$

$\Rightarrow dx = \frac{du}{-3x^{-4}}$

$\Rightarrow \int \cancel{3x^{-4}}\, u^5 \cdot \frac{du}{-\cancel{3x^{-4}}}$

$= -\int u^5 du$

$= -\frac{u^6}{6} + C$

$= \boxed{-\frac{\left(\frac{1}{x^3}+4\right)^6}{6} + C}$

(c) $\int \frac{x^2 + x - e^{2x}}{3}\,dx$

$$= \int \frac{x^2}{3} + \frac{x}{3} - \frac{e^{2x}}{3}\,dx$$

$$= \boxed{\frac{x^3}{9} + \frac{x^2}{6} - \frac{e^{2x}}{6} + C}$$

(d) $\int_0^{\ln 3} \frac{3e^{2x}}{(1+e^{2x})^2}\,dx$

$$u = 1 + e^{2x}$$

$$du = 2e^{2x}\,dx$$

$$\Rightarrow dx = \frac{du}{2e^{2x}}$$

$$\Rightarrow \int_{x=0}^{\ln 3} \frac{3\cancel{e^{2x}}}{u^2} \cdot \frac{du}{2\cancel{e^{2x}}}$$

$$= \frac{3}{2}\int_{x=0}^{\ln 3} u^{-2}\,du$$

$$= -\frac{3}{2}u^{-1}\Big|_{x=0}^{\ln 3}$$

$$= -\frac{3}{2}(1+e^{2x})^{-1}\Big|_0^{\ln 3}$$

$$-\frac{3}{2}(1+e^{2\ln 3})^{-1} - -\frac{3}{2}(1+e^0)^{-1}$$

$$= \boxed{-\frac{3}{2}(10)^{-1} + \frac{3}{2}(2)^{-1}}$$

OR

$$= -\frac{3}{20} + \frac{3}{4}$$

$$= \boxed{\frac{3}{5}}$$

(e) $\int (x-1)(x+1)e^{x^3-3x}\,dx$

$= \int (x^2-1)e^{x^3-3x}\,dx$

$u = x^3-3x$

$du = (3x^2-3)\,dx$

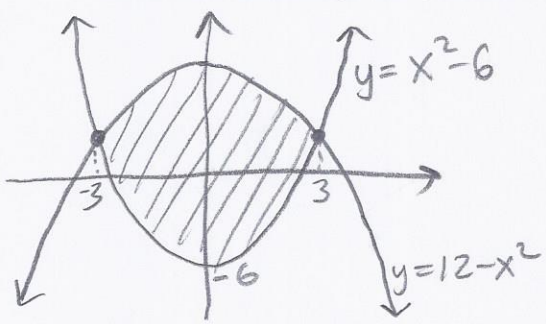
$\Rightarrow dx = \dfrac{du}{3x^2-3} = \dfrac{du}{3(x^2-1)}$

$\Rightarrow \int \cancel{(x^2-1)}e^u \cdot \dfrac{du}{3\cancel{(x^2-1)}}$

$= \frac{1}{3}\int e^u\,du$

$= \frac{1}{3}e^u + C = \boxed{\frac{1}{3}e^{x^3-3x}+C}$

2. (30 points) Find the area of the region bounded by the curves $y = x^2 - 6$ and $y = 12 - x^2$. Include a sketch of the region in your answer and shade the required area.



Intersections

$x^2-6 = 12-x^2$

$\Rightarrow 2x^2-18=0$

$\Rightarrow 2(x^2-9)=0$

$x = \pm 3$

$A = \int_{-3}^{3} 12-x^2-(x^2-6)\,dx$

$= \int_{-3}^{3} -2x^2+18\,dx$

$= -\frac{2x^3}{3}+18x \Big|_{-3}^{3}$

$= -\frac{2(3)^3}{3}+18(3) - \left(+\frac{2(3)^3}{3}-18(3)\right)$

$= -\frac{4(3)^{\cancel{3}\,2}}{\cancel{3}} + 36(3)$

$= -4(9)+12(9)$

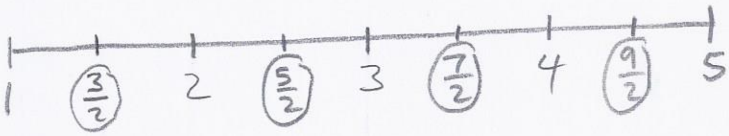
$= 8(9) = \boxed{72}$

Alternatively

$A = 2\int_{0}^{3} 12-x^2-(x^2-6)\,dx$

3. (20 points) Use Riemann sums to approximate the net area under the curve $y = 25 - x^2$ on the interval $1 \leq x \leq 5$ using $n = 4$ subintervals and *midpoints*. You may leave your answer as a sum of fractions.

$$\Delta x = \frac{5-1}{4} = 1$$



$$\Rightarrow A \approx M_4 = \Delta x \left(f\left(\tfrac{3}{2}\right) + f\left(\tfrac{5}{2}\right) + f\left(\tfrac{7}{2}\right) + f\left(\tfrac{9}{2}\right) \right)$$

$$= 1\left(25 - \left(\tfrac{3}{2}\right)^2 + 25 - \left(\tfrac{5}{2}\right)^2 + 25 - \left(\tfrac{7}{2}\right)^2 + 25 - \left(\tfrac{9}{2}\right)^2 \right)$$

$$= \boxed{25 - \frac{9}{4} + 25 - \frac{25}{4} + 25 - \frac{49}{4} + 25 - \frac{81}{4}}$$

Bonus Problems: (You must complete all problems in the actual test to be eligible).

1. (10 points) The velocity of a moving particle is given by $v(t) = 6t^2 - 4$, where t is time in seconds, and velocity is measured in meters per second. Find its position function, $s(t)$, if you know that $s(2) = 10$.

$$s(t) = \int v(t)\,dt$$
$$= \int 6t^2 - 4\,dt$$
$$= 2t^3 - 4t + C$$
$$s(2) = 10 \Rightarrow 10 = 2(2)^3 - 4(2) + C$$
$$\Rightarrow 10 = 8 + C$$
$$\Rightarrow C = 2$$
$$\Rightarrow \boxed{s(t) = 2t^3 - 4t + 2}$$

2. (10 points) A particle moves along the circle $x^2 + y^2 = 8$. As the particle passes through the point (2,2), its x-coordinate is *decreasing* at a rate of 2 units per second. At what rate is the y-coordinate changing at this instant? Is it increasing or decreasing?

$$x^2 + y^2 = 8$$
$$\Rightarrow \cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = 0$$
$$\Rightarrow (2)(-2) + (2)\frac{dy}{dt} = 0$$
$$\Rightarrow \boxed{\frac{dy}{dt} = 2}$$

The y-coordinate is increasing at 2 units/sec


When did you become an expert in integration?
Last night.

Math 205 Test 4 Grading Guidelines

Problem 1

(a)

- 4 points to figure out the correct substitution and find du and dx
- 4 points to get to the new integral, $\frac{2}{5}\int u^{-\frac{1}{3}}du$
- 1 point to integrate
- 1 point for final answer as a function of x
- 2 points are deducted if student forgets the +C.

(b) Similar to (a)

(c)

- 6 points for rewrite
- 4 points for final answer
- 2 points are deducted if +C is forgotten
- No credit given if student tried substitution or something crazy.

(d)

- 4 points for correct substitution and to find du and dx
- 2 points to get to the new integral, $\frac{3}{2}\int u^{-2}du$
- 2 points to get the answer just before plugging in the limits
- 1 point for applying the fundamental theorem of calculus and plugging in the limits correctly
- 1 point for the boxed answer. Either answer is fine.

(e) Similar to (a)

Problem 2.

- 5 points for correct sketch, labeled and shaded
- 5 points to find the correct intersections
- 5 points to set up the integral correctly, with the right limits
- 5 points to integrate and bring the problem to just before plugging in the limits
- 5 points to plug in the numbers correctly
- 5 points to simplify to the final answer of 72.

Problem 3.

- 5 points to figure out Δx
- 5 points to find the midpoints, whether or not the interval diagram is used to do so. Work must be shown though
- 5 points to plug into the area formula
- 5 points for final answer as a sum of fractions

- If a student goes all the way, adds the numbers and gets the final answer incorrect, 4 points will be deducted.

Bonus.

Problem 1:
- 4 points to figure out that $s(t) = \int v(t)dt$
- 2 points to integrate correctly
- 2 points to find C
- 2 points for final answer.

Problem 2:
- 4 points to differentiate the equation implicitly correctly
- 4 points to plug in the knowns correctly
- 1 point to find $\frac{dy}{dt}$
- 1 point to state it is increasing.

Required knowledge:

For this exam, the student needs to: know how to use a Riemann sum to approximate an area. Also: Know how to compute definite and indefinite integrals, which may involve integration by substitution. The student must also be able to find the area bounded between two curves using integration and the fundamental theorem of calculus.

Optional Knowledge:

To gain bonus points, the student needed to know how to apply integration to solve a position-velocity-acceleration problem. The student also needed to recall how to solve a related rates problem—a topic on a previous exam.