**Math 205 Test 1**
September 28, 2016

Name: JHEVON SMITH

**Note that both sides of each page may have printed material.**

**Instructions:**

1. Read the instructions.

2. Panic!!! Kidding, don't panic! I repeat, **do NOT panic**! Don't look down, while you're at it.

3. Complete all problems in the actual test. Bonus problems are, of course, optional. And they will only be counted if all other problems are attempted.

4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers.

5. Write neatly so that I am able to follow your sequence of steps and box your answers.

6. Read through the exam and complete the problems that are easy (for you) first!

7. Calculators are NOT allowed. And you are NOT allowed to use notes, or other aids—including, but not limited to, divine intervention/inspiration, the internet, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.

8. In fact, **cell phones should be out of sight!**

9. Use the correct notation and write what you mean! $x^2$ and $x2$ are not the same thing, for example, and I will grade accordingly.

10. Other than that, have fun and good luck!

    May the force be with you. But you can't ask it to help you with your test.

1. (5 points each) Simplify the following:

(a) $\frac{5(3x^2+1)^4(6x)(2x^3-1)^4-4(2x^3-1)^3(6x^2)(3x^2+1)^5}{(2x^3-1)^8}$

$$= \frac{6x(3x^2+1)^4\cancel{(2x^3-1)^3}[5(2x^3-1) - 4x(3x^2+1)]}{(2x^3-1)^{\cancel{8}\,5}}$$

$$= \frac{6x(3x^2+1)^4[10x^3-5-12x^3-4x]}{(2x^3-1)^5}$$

$$= \boxed{\frac{-6x(3x^2+1)^4(2x^3+4x+5)}{(2x^3-1)^5}}$$

(b) $\ln\sqrt{\frac{3e^x\sqrt{x}}{x^2(x-1)^4}} = \frac{1}{2}\ln\left(\frac{3e^x x^{1/2}}{x^2(x-1)^4}\right)$

$$= \frac{1}{2}\left(\ln 3 + \ln e^x + \tfrac{1}{2}\ln x - 2\ln x - 4\ln(x-1)\right)$$

$$= \boxed{\frac{1}{2}\left(\ln 3 + x - \tfrac{3}{2}\ln x - 4\ln(x-1)\right)}$$

(c) $e^{\left(\frac{1}{2}\right)\ln y - 5\ln(3x) - 3}$

$$= e^{\ln y^{1/2}} \cdot e^{\ln(3x)^{-5}} \cdot e^{-3}$$

$$= y^{1/2} \cdot (3x)^{-5} \cdot e^{-3}$$

$$= \boxed{\frac{y^{1/2}}{e^3(3x)^5}}$$

(d) $\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$

$$= \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} \cdot \frac{x^2(x+h)^2}{x^2(x+h)^2}$$

$$= \frac{x^2 - (x+h)^2}{h(x^2)(x+h)^2}$$

$$= \frac{\cancel{x^2} - \cancel{x^2} - 2xh - h^{\cancel{2}}}{\cancel{h}(x^2)(x+h)^2}$$

$$= \boxed{\frac{-2x-h}{x^2(x+h)^2}}$$

2. (a) (8 points) Find the equation of the line that passes through $(1, -2)$ that is perpendicular to $3x - 2y = 4$. $\Rightarrow m_1 = 3/2 \Rightarrow m = -2/3$

Using $y - y_1 = m(x - x_1)$, we get

$$\boxed{y + 2 = -\frac{2}{3}(x - 1)}$$

(c) (6 points each) Find and simplify the indicated compositions, given $f(x) = \sqrt{2x^2 + 4}$ and $g(x) = \sqrt{x^2 - 4}$, and state the domains of each composite function:

(i) $f \circ g = f(g)$

$= \sqrt{2(\sqrt{x^2-4})^2 + 4}$

$= \boxed{\sqrt{2x^2 - 4}}$

dom$(f \circ g)$: $2x^2 - 4 \geqslant 0$

$x^2 \geqslant 2$


dom$(g)$: $x^2 - 4 \geqslant 0$

$x^2 \geqslant 4$


$\Rightarrow$ Overlap: $(-\infty, -2) \cup (2, \infty)$

Domain of $f \circ g$: $\boxed{(-\infty, -2) \cup (2, \infty)}$

(ii) $g \circ f = g(f)$

$= \sqrt{(\sqrt{2x^2+4})^2 - 4}$

$= \boxed{\sqrt{2x^2}}$

dom $g(f)$: $(-\infty, \infty) \to 2x^2 \geqslant 0$ always.

dom$(f)$: $2x^2 + 4 \geqslant 0$

$\hookrightarrow$ always true!

$(-\infty, \infty)$

Overlap: $(-\infty, \infty)$

Domain of $g \circ f$: $\boxed{(-\infty, \infty)}$

3. (a) (4 points) Let $f(x)$ be a function, state its difference quotient.

$$\boxed{\frac{f(x+h)-f(x)}{h}} \quad \text{OR} \quad \boxed{\frac{f(b)-f(a)}{b-a}}$$

(b) (8 points each) Find and simplify the difference quotient of the following functions:

(i) $f(x) = 5 - x^2$

$$\frac{f(x+h)-f(x)}{h} = \frac{5-(x+h)^2-(5-x^2)}{h}$$

$$= \frac{\cancel{5}-\cancel{x^2}-2x\cancel{h}-h^{\cancel{2}}-\cancel{5}+\cancel{x^2}}{\cancel{h}}$$

$$= \boxed{-2x-h}$$

OR

$$\frac{f(b)-f(a)}{b-a} = \frac{5-b^2-(5-a^2)}{b-a}$$

$$= \frac{\cancel{5}-b^2-\cancel{5}+a^2}{b-a}$$

$$= \frac{(\cancel{a-b})(a+b)}{\cancel{b-a}\,(-1)}$$

$$= \boxed{-(a+b)}$$

(ii) $f(x) = 2 + x - \frac{3}{x}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\cancel{2}+\cancel{x}+h-\frac{3}{x+h}-(\cancel{2}+\cancel{x}-\frac{3}{x})}{h}$$

$$= \frac{h - \frac{3}{x+h} + \frac{3}{x}}{h}$$

$$= \frac{\cancel{h}}{\cancel{h}} + \frac{\frac{3}{x} - \frac{3}{x+h}}{h} \cdot \frac{x(x+h)}{x(x+h)}$$

$$= 1 + \frac{3(\cancel{x}+h)-\cancel{3x}}{\cancel{h}x(x+h)}$$

$$= \boxed{1 + \frac{3}{x(x+h)}}$$

OR

$$\frac{f(b)-f(a)}{b-a} = \frac{\cancel{2}+b-\frac{3}{b}-(\cancel{2}+a-\frac{3}{a})}{b-a}$$

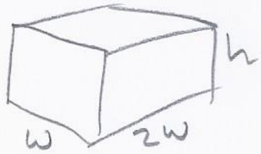
$$= \frac{(b-a) + \frac{3}{a} - \frac{3}{b}}{b-a}$$

$$= \frac{\cancel{b-a}}{\cancel{b-a}} + \frac{\frac{3}{a}-\frac{3}{b}}{b-a} \cdot \frac{ab}{ab}$$

$$= 1 + \frac{3b-3a}{(b-a)ab}$$

$$= \boxed{1 + \frac{3}{ab}}$$

4. (a) (5 points) A closed rectangular box with volume 8 $ft^3$ has length twice the width. Express the height of the box, $h$, as a function of the width, $w$.




We know Volume $= lwh$

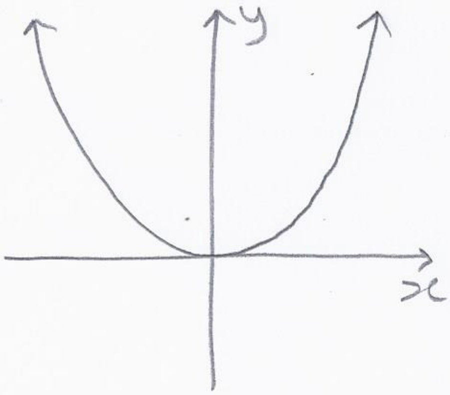
$= 2w \cdot w \cdot h = 8$

$\Rightarrow 2w^2h = 8$

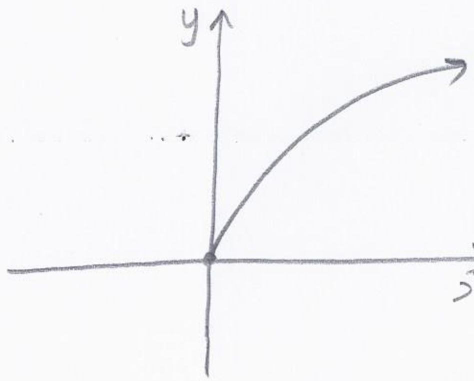
$\Rightarrow \boxed{h = \frac{4}{w^2}}$

(b) (3 points each) Sketch the graphs of the following:

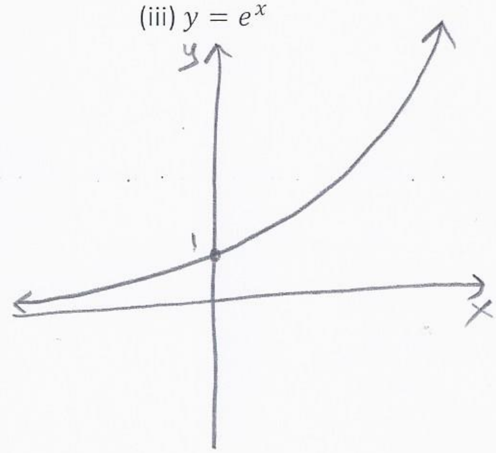
(i) $y = x^2$



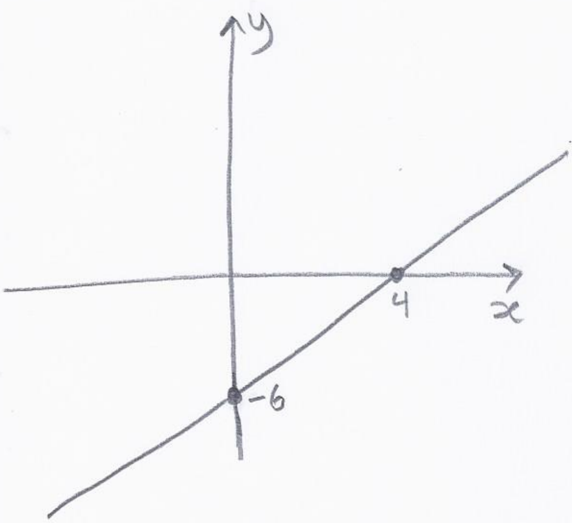

(ii) $y = \sqrt{x}$



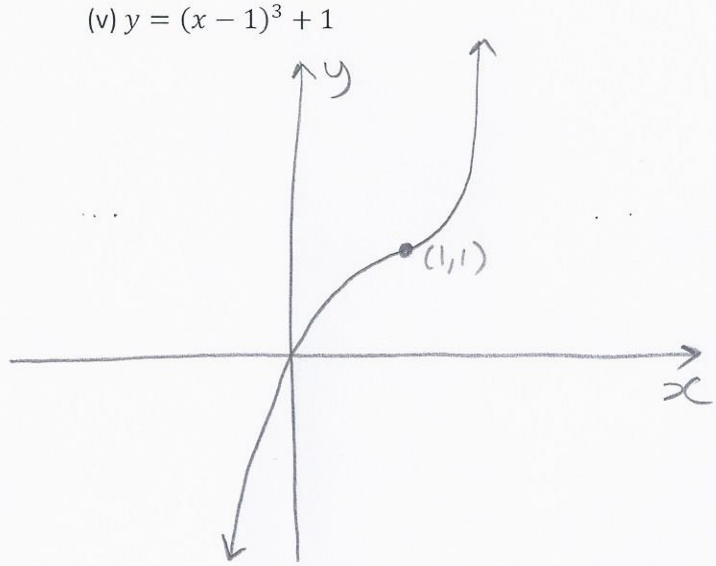

(iii) $y = e^x$



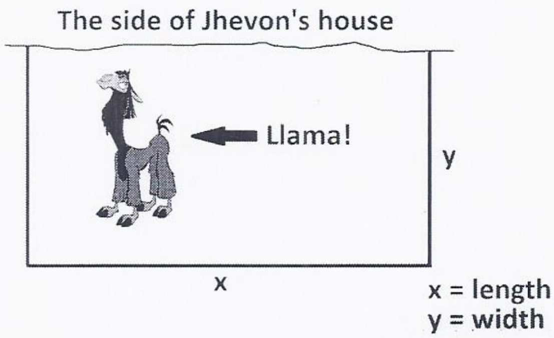

(iv) $3x - 2y = 12$




(v) $y = (x-1)^3 + 1$



5. (10 points) Jhevon's llamas are out of control, and he decides to build a rectangular fence enclosure in his backyard to keep them out of trouble. He decides to use 100 feet of fencing to make three sides of the fence, with the back of his house forming the fourth side (because who has time to make a four sided enclosure these days anyway?). Figure out **what dimensions the fence must have** so that his llamas have the most room to frolic. Hint: Use the given diagram. Describe the area as a function of the length of one of the sides, and then find the value so that this area function is as large as possible. Use this to figure out the dimensions.




We know Perimeter = 100

$\Rightarrow x + 2y = 100$

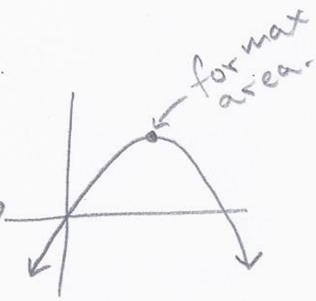
$\hookrightarrow x = 100 - 2y$

Most room $\Rightarrow$ largest area.

$\therefore A = xy$

$\Rightarrow A = (100-2y)y$

$= 100y - 2y^2$




We need the vertex!

$y = \frac{-100}{2(-2)} = 25 \text{ ft}$

$x = 100 - 2y = 50 \text{ ft}.$

$\Rightarrow$ Dimensions: $x = 50\text{ft},\ y = 25\text{ft}$ or $50 \times 25$

(b) (5 points each) Solve the following equations:

(i) $e^{x^2-3} - 7 = 0$

$\Rightarrow e^{x^2-3} = 7$

$\Rightarrow x^2 - 3 = \ln 7$

$\Rightarrow \boxed{x = \pm\sqrt{\ln 7 + 3}}$

(ii) $\ln(x+1)^2 = -4$

$\Rightarrow (x+1)^2 = e^{-4}$

$\Rightarrow x + 1 = \pm\sqrt{e^{-4}}$

$\Rightarrow \boxed{x = -1 \pm \sqrt{e^{-4}}}$

OR

$\boxed{x = -1 \pm e^{-2}}$

**Bonus Problems (4 points each problem):**

1. Compute the following limits:

$(i)\ \lim_{x\to -1} \frac{x^2+4x+3}{x^2-x-2} = \lim_{x\to -1} \frac{(x+3)\cancel{(x+1)}}{(x-2)\cancel{(x+1)}}$

$= \frac{2}{-3}$

$= \boxed{-\frac{2}{3}}$

$(ii)\ \lim_{x\to 0^+} \frac{3x^2+3}{5x^2-4} = \frac{3(0)^2+3}{5(0)^2-4}$

$= \boxed{-\frac{3}{4}}$

2. Use the limit definition to find the derivative of $f(x) = \frac{x}{x+1}$.

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{\frac{x+h}{x+h+1} - \frac{x}{x+1}}{h} \cdot \frac{(x+1)(x+h+1)}{(x+1)(x+h+1)}$

$= \lim_{h\to 0} \frac{(x+h)(x+1) - x(x+h+1)}{h(x+1)(x+h+1)}$

$= \lim_{h\to 0} \frac{\cancel{x^2}+\cancel{x}+\cancel{xh}+h-\cancel{x^2}-\cancel{xh}-\cancel{x}}{h(x+1)(x+h+1)}$

$= \lim_{h\to 0} \frac{1}{(x+1)(x+h+1)} = \boxed{\frac{1}{(x+1)^2}}$

3. Find derivatives, show your work:

(i) $\frac{d}{dx}\left(\frac{(3x^2+1)^5}{(2x^3-1)^4}\right) =$

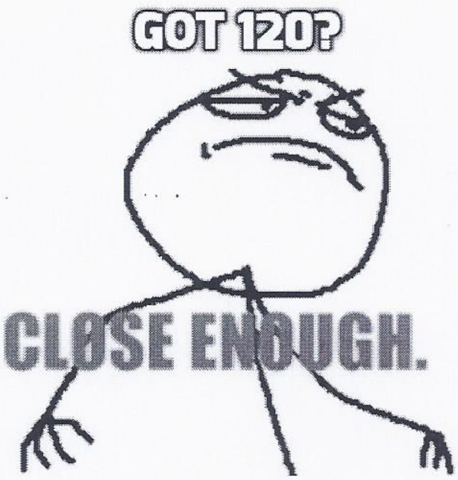
$\frac{5(3x^2+1)^4(6x)(2x^3-1)^4 - 4(2x^3-1)^3(6x^2)(3x^2+1)^5}{(2x^3-1)^8}$

By, the Quotient Rule.
This is the same as
problem 1(a), so...

$\boxed{\frac{-6x(3x^2+1)^4(2x^3+4x+5)}{(2x^3-1)^5}}$

(ii) $\frac{d}{dx}(3x^3 + e^{4x}) =$

$= \boxed{9x^2 + 4e^{4x}}$


GOT 120?
CLOSE ENOUGH.

# Math 205 ST Test 1 Grading Guidelines

**Problem 1**

Part a: 3 points for knowing to factor out the common term from the numerator and doing this correctly. 1 point for the intermediate simplification. 1 point for final answer. Leaving it as $\frac{6x(3x^2+1)^4(-2x^3-4x-5)}{(2x^3-1)^5}$ is OK.

Part b: 3 points to get to line 2 in the solutions. 2 points to finish and get to line 3. Multiplying out the ½ is OK.

Part c: 3 points for knowing to split up as a product (or divisions) with base $e$. 2 points to get to the final correct answer.

Part d: Knowing to multiply by the $\frac{LCD}{LCD}$ to simplify is worth 3 points. Equivalently, a student could find the common denominator in the numerator, combine the fractions, then flip and multiply. That will bring them to line 2 of the solutions. This must be done correctly. 2 points to simplify correctly and get to the final answer.

**Problem 2**

Part a: 4 points for finding the correct $m$ to use. 2 points for identifying the correct $(x_1, y_1)$ and knowing what formula to plug in to. The student may use the formula $y = mx + b$, though that would be harder. 2 points to get to the final answer of $y + 2 = -\frac{2}{3}(x - 1)$ by using the point-slope form. It is fine, but not necessary, to solve for $y$ to get $y = -\frac{2}{3}x - \frac{4}{3}$.

Part b: (mistakenly labelled part c because MS Word decided to spazz out) 3 points for finding the composite function. 2 points to realize that you need to find the intersection of the domain of the composite and the inside function. This is shown on the side in the solutions. The student does not need to show this. 1 point for the final answer. If the final answer is correct for both, but the student did not show how to get the domain, this is OK, they still get full credit.

**Problem 3**

Part a: All or nothing! I've emphasized this formula so much in class. The student should know it by heart by now! 4 points for the correct formula, if anything is off, 0 points. The student is allowed to use either version. Both versions are in the solutions.

Part b: For both parts b(i) and b(ii) the student can use any version of the difference quotient. Solutions for both are in the solutions file. 2 points for the final correct answer, the rest of the points are distributed evenly among the other lines of the solution. One thing to watch out for here is blasphemy! Remember that, for example, $(x + h)^2 = x^2 + 2xh + h^2$. If the student writes $(x + h)^2 = x^2 + h^2$, well, then it's all over.

**Problem 4**

Part a: 3 points to remember the volume formula is $V = lwh$ and recognizing that $l = 2w$ here, since we know the length is twice the width. 1 point for setting up the right equation, $2w^2h = 8$. 1 point for the final answer of $h = 4/w^2$

Part b: 3 points for each graph. For graphs (iii) and (iv), the student should indicate the intercepts and should receive 2 points if they are not indicated.

**Problem 5**

Part a: Follow the hints. Finding the perimeter equation would be the first step, 3 points for doing this. Figuring out the area formula in one variable (by plugging in the perimeter equation), 3 points. Finding the vertex, 2 points. Final answer, 2 points. This problem is perhaps the most challenging in the test, but very doable. The problems from the review from the textbook and the kind of analysis we did for the hotdog stand problem in lecture 2 will help you out here.

Part b: For each part here, b(i) and b(ii), 1 point for the correct final answer. The remaining 4 points distributed evenly over the other lines of the solution.

**Bonus**

1.(i) 3 points to factor and simplify, 1 point for final answer.

1.(ii) Just plug in…hmm, all or nothing here. Once you see what to do, there should be no excuse for getting it incorrect.

2. 1 point for writing out the formula. 2 points for simplifying and plugging in the limit. 1 point for final answer. (2 points will be deducted if the student does not write $\lim\limits_{h \to 0}$ as often as they should. It must be there until $h = 0$ is plugged in.

3.(i) The student would need to know the quotient rule for this. 3 points to apply it correctly, 1 point for final answer. (Student would hopefully recognize that this is the same as problem 1(a) and so that would save time.

3.(ii) The student needs to know the power rule $\frac{d}{dx}x^n = nx^{n-1}$ and the rule $\frac{d}{dx}e^u = u'e^u$. All or nothing here, there's nothing to do once those rules are applied.

**Required knowledge:**

For this exam, the student needs to: know algebra (of course! That'll be true for every test), how to factor, how to simplify, including simplifying rational functions. How to graph common graphs. How to find the equations of lines. How to find composite functions and domains of functions. Have the difference quotient memorized and know how to apply it to a function. Set up functions, with the help of diagrams. Solve exponential and log equations. The student should pay attention to what areas he/she is weak in and work on that for the next test.

**Optional Knowledge:**

To gain bonus points, the student needed to know how to compute limits. How to find the derivative of a function using the limit definition of a derivative. How to apply the quotient rule, the power rule, the chain rule, and the derivative of exponential rule.