Math 205 Quiz 8

October 24, 2016

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. Let $f(x)$ and $g(x)$ be differentiable functions of x, c a constant. Complete the following formulas. (You may use f' and g' as shorthand):

   (a) $\frac{d}{dx}(x^n) = nx^{n-1}$ (b) $\frac{d}{dx}e^u = u'e^u$ (c) $\frac{d}{dx}(a^u) = u'a^u \ln a$

   (d) $\frac{d}{dx}\ln u = \frac{u'}{u}$ (e) $\frac{d}{dx}(f(x)\cdot g(x)) = f'g + fg'$

   (f) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'g - fg'}{g^2}$ (g) $\frac{d}{dx}f(g(x)) = f'(g(x))\cdot g'(x)$

2. Define $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ (using limits)

3. Simplify: $\ln\left(\frac{\sqrt{x}(x+1)}{x^2e^x}\right) = \frac{1}{2}\ln x + \ln(x+1) - 2\ln x - x$

4. Differentiate the following (you don't have to simplify):

   (a) $\frac{d}{dx}\ln\left(\frac{\sqrt{x}(x+1)}{x^2e^x}\right) = \frac{1}{2x} + \frac{1}{x+1} - \frac{2}{x} - 1$ (b) $\frac{d}{dx}[\ln x^3 + (\ln x)^3] = \frac{3}{x} + 3(\ln x)^2\cdot\frac{1}{x}$

   (c) $\frac{d}{dx}\frac{x^2}{x^2+1} = \frac{2x}{(x^2+1)^2}$ (d) $\frac{d}{dx}x\sqrt{x^2-1} = (x^2-1)^{1/2} + \frac{1}{2}x(x^2-1)^{-1/2}(2x) = \frac{2x^2-1}{(x^2-1)^{1/2}}$

   (e) $\frac{d}{dx}\frac{5e^{2x}+3}{4} = \frac{5}{2}e^{2x}$ (f) $\frac{d}{dx}\sqrt[4]{2x^3+7x} = \frac{1}{4}(2x^3+7x)^{-3/4}(6x^2+7)$

   (g) $\frac{d}{dx}(x+1)^5(3x-1)^7 = 5(x+1)^4(3x-1)^7 + (x+1)^5\cdot 7(3x-1)^6(3)$ (h) $\frac{d}{dx}\frac{x^2+1}{x^2} = -2x^{-3}$

5. The position of a particle after time t seconds is given by $s(t) = t^4 - 3t^2 + 2t - 3$. What is:

   (a) The particle's acceleration function? $a(t) = 12t^2 - 6$

   (b) The particle's velocity after 2 seconds? 22 units/sec

6. Find y' if : $4x^2 + 2xy - 3x^2y^3 + 4 = 7 - \ln x$: $y' = \frac{6xy^3 - 8x - 2y - \frac{1}{x}}{2x - 9x^2y^2}$

Bonus:

1. If $y = x^x$, then $y' = x^x(\ln x + 1)$

2. For the function $f(x)$, state the linear approximation formula for $f(x)$ at the point where $x = a$:

   $f(x) \approx f(a) + f'(a)(x-a)$

3. Find, by any method, $\frac{d}{dx}\frac{\sqrt{x}(x+1)}{x^2e^x} = y\left(\frac{1}{2x} + \frac{1}{x+1} - \frac{2}{x} - 1\right)$ *

   * $y = \frac{\sqrt{x}(x+1)}{x^2e^x}$