Math 205 Test 3
July 18, 2016

Name: JHEVON SMITH

Note that both sides of each page may have printed material.

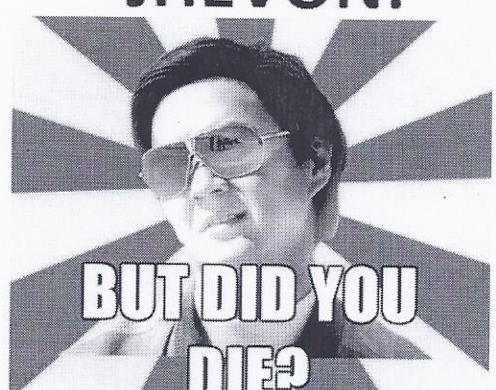
Instructions:

1. Read the instructions.
2. Panic!!! Kidding, don't panic! I repeat, do NOT panic!
3. Complete all problems in the actual test. Bonus problems are, of course, optional. And they will only be counted if all other problems are attempted.
4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers.
5. Write neatly so that I am able to follow your sequence of steps and box your answers.
6. Read through the exam and complete the problems that are easy (for you) first!
7. Scientific calculators are needed, but you are NOT allowed to use notes, or other aids—including, but not limited to, divine intervention/inspiration, the internet, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.
8. In fact, **cell phones should be out of sight!**
9. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.
10. Other than that, have fun and good luck!

STUDENT:

Jhevon chill! You make this class too hard. My GPA!!!

JHEVON:



It's tough love. Now get focused and get this done. Hooraah!

1. (4 points each part) A population $P(t)$ doubles every 3 years. In 1990, the population was 4 million. Assume exponential growth and that we start measuring the population in 1990.

(a) Find the differential equation satisfied by $P(t)$. Also include the initial condition.

$r = \frac{\ln 2}{3}$

$\Rightarrow \boxed{P' = \frac{\ln 2}{3} P, \quad P(0) = 4 \text{ million.}}$

(b) Find and simplify $P(t)$

$P = P_0 e^{rt}$

$\Rightarrow \boxed{P = 4e^{\frac{\ln 2}{3} t} \text{ in millions}}$

OR $P = 4e^{\frac{\ln 2}{3} t}$

$= 4\left(e^{\ln 2}\right)^{t/3}$

$\boxed{P = 4 \cdot 2^{t/3} \text{ in millions}}$

(c) What is the population's size in 1992?

We want $P(2)$

$\boxed{P(2) = 4e^{\frac{\ln 2}{3}(2)}}$ million

or equivalently, $\boxed{P(2) = 4 \cdot 2^{2/3}}$ million

(d) After how long will the population be 10 million?

We want t so that $P(t) = 10$

$\Rightarrow 4e^{\frac{\ln 2}{3} t} = 10$

$\Rightarrow e^{\frac{\ln 2}{3} t} = 5/2$

$\Rightarrow \frac{\ln 2}{3} t = \ln \frac{5}{2}$

$\Rightarrow \boxed{t = \frac{3 \ln \frac{5}{2}}{\ln 2}}$ years

(e) How fast is the population growing when it reaches 6 million?

Using $P' = rP$

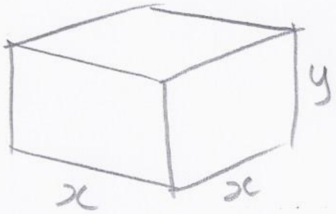
$\Rightarrow P'(6) = \frac{\ln 2}{3} \cdot 6$

$= \boxed{2 \ln 2}$ million/year

2. (20 points) A closed rectangular box with a square base and volume 12 cubic feet is to be constructed using two different types of materials. The top is made of metal costing $2 per square foot, and the remaining sides and base is made of wood costing $1 per square foot. Find the dimensions of the box for which the cost of construction is minimized.

① Read!

②



③ Constraint: $V = x^2 y = 12$

$\Rightarrow y = \frac{12}{x^2}$

Objective: $C = \underbrace{2x^2}_{\text{top}} + \underbrace{x^2}_{\text{bottom}} + \underbrace{4xy}_{\text{4 sides}}$

$\Rightarrow C = 3x^2 + 4xy$

④ $C = 3x^2 + 4x\left(\frac{12}{x^2}\right)$

$\Rightarrow C = 3x^2 + 48x^{-1}$

⑤ For min C, set $C' = 0$

$\Rightarrow 6x - 48x^{-2} = 0$

$\Rightarrow x - 8x^{-2} = 0$

$\Rightarrow x^3 = 8$

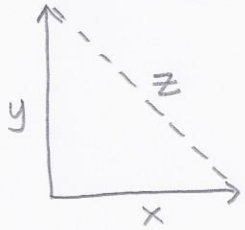
$\Rightarrow x = 2$

$\Rightarrow y = \frac{12}{2^2} = 3$

⑥ $\boxed{\text{The dimensions for min Cost are } 2 \times 2 \times 3}$

3. (20 points) Two cars drive away from the same point. One heads north at 40 mph. The other drives east at 30 mph. After 1 hour, at what rate is the distance between them changing?

Step ①/②



Let y be the distance the north car drives
Let x " " " " east " "
Let z be the distance between both cars.

Know: $\frac{dy}{dt} = 40$, $\frac{dx}{dt} = 30$

After 1 hour, $x = 30$, $y = 40 \Rightarrow z = 50$ (Pythagoras' thm or 3-4-5 Δ).

Want: $\frac{dz}{dt}$ after 1 hour.

③ By Pythagoras

$$x^2 + y^2 = z^2$$

④ $$\Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$\Rightarrow \frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z}$$

⑤ $$\Rightarrow \frac{dz}{dt} = \frac{30(30) + 40(40)}{50}$$

$$= \frac{\cancel{100}^{2}(9+16)}{\cancel{50}_{1}}$$

$$= 50$$

⑥ $\boxed{\text{The distance between them is increasing at a rate of 50 mph}}$

4. (20 points) For the function $f(x) = \frac{x^3}{x^2-3}$ find (provided they exist) the domain, intercepts, asymptotes, local extrema, inflection point(s), intervals of increasing and decreasing, and intervals of concavity. You may assume, without verification, that $f'(x) = \frac{x^2(x^2-9)}{(x^2-3)^2}$ and $f''(x) = \frac{6x(x^2+9)}{(x^2-3)^3}$.

Domain: $(-\infty,-\sqrt{3})\cup(-\sqrt{3},\sqrt{3})\cup(\sqrt{3},\infty)$

Intercepts: x-int: $x=0$
y-int: $y=0$

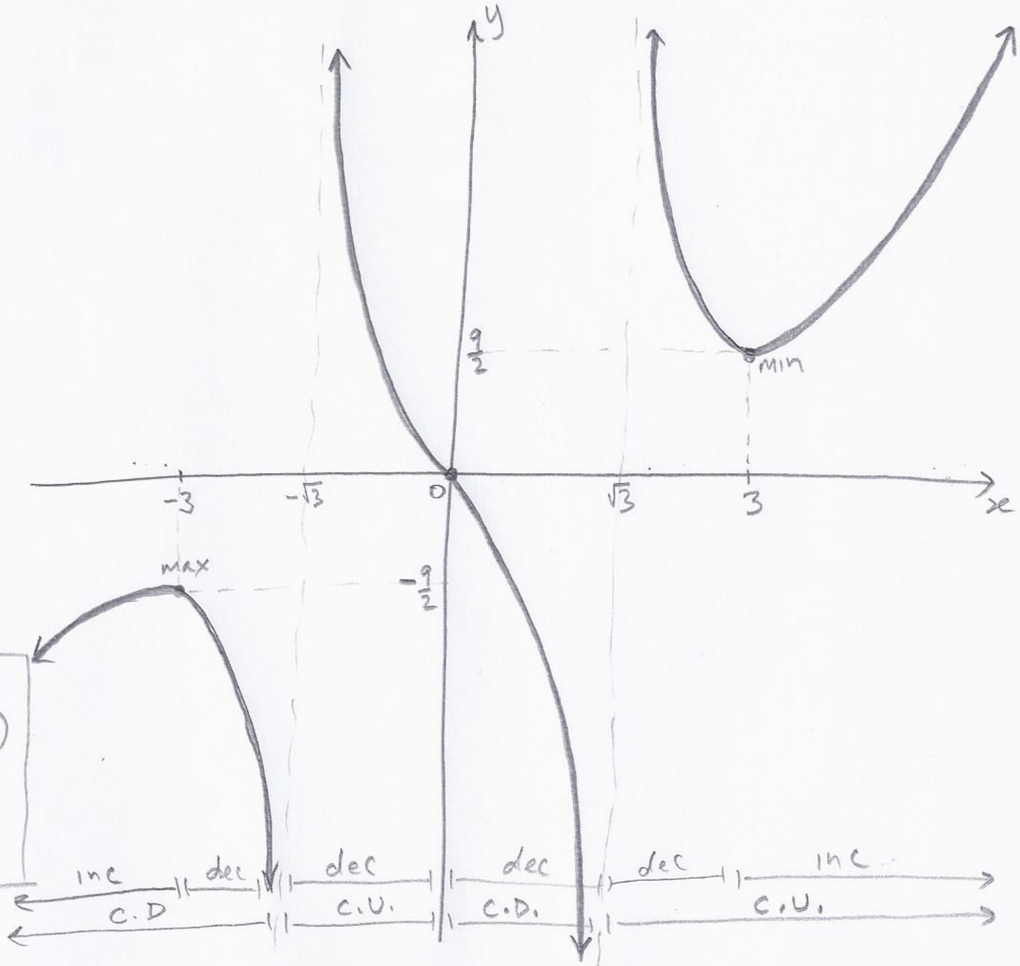
Asymptotes: VA
$x^2-3=0$
$\boxed{x=\pm\sqrt{3}}$ V.A.s.

HA
$\lim_{x\to\infty} f(x)=\infty$
$\lim_{x\to-\infty} f(x)=-\infty$
$\boxed{\text{No HA!}}$

Inc/Dec/Max/Min :
$x=0,\pm 3,\pm\sqrt{3}$

(+) (−) (−) (−) (−) (+) f'
-3 $-\sqrt{3}$ 0 $\sqrt{3}$ 3
max min

Increasing: $(-\infty,-3)\cup(3,\infty)$
Decreasing: $(-3,-\sqrt{3})\cup(-\sqrt{3},0)\cup(0,\sqrt{3})\cup(\sqrt{3},3)$
max: $(-3,-9/2)$
min: $(3, 9/2)$



Concavity / Inflections
$x=0,\pm\sqrt{3}$

(−) (+) (−) (+) f''
$-\sqrt{3}$ 0 $\sqrt{3}$
Inf

Concave up: $(-\sqrt{3},0)\cup(\sqrt{3},\infty)$
Concave down: $(-\infty,-\sqrt{3})\cup(0,\sqrt{3})$
Inflection: $(0,0)$

5. (20 points) Let $f(x) = 2x^3 - 3x^2 - 12x + 1$, find the absolute maximum and minimum of $f(x)$ on the interval $[-2,1]$.

$f' = 6x^2 - 6x - 12$

For critpts:

$6x^2 - 6x - 12 = 0$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0$

$\Rightarrow \cancel{x=2}\,,\ x = -1$

reject.
Not in $[-2,1]$.

Check crit. pt.:

$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$
$= 8$

Check endpoints:

$f(-2) = 2(-2)^3 - 3(-2)^2 - 12(-2) + 1$
$= -3$

$f(1) = 2(1)^3 - 3(1)^2 - 12(1) + 1$
$= -12$

$\Rightarrow$ abs max: $f(-1) = 8$

abs min: $f(1) = -12$

Bonus Problems: (You must complete all problems in the actual test to be eligible).

1. (5 points) Using Riemann sums with 4 subintervals, approximate the area under $f(x) = x^2 + 1$ on the interval $[-1,3]$ using left hand endpoints. Show all your work.

$\Delta x = \frac{3-(-1)}{4} = 1$

-1 0 1 2 3

$A \approx L_4 = \Delta x(f(-1)+f(0)+f(1)+f(2))$
$= (-1)^2+1+0^2+1+1^2+1+2^2+1$
$= \boxed{10}$

2. (5 points) Find the exact area under $f(x) = x^2 + 1$ on $[-1,3]$. Is your approximation in problem 1 an over or underestimate?

$\int_{-1}^{3} x^2+1\,dx = \frac{x^3}{3}+x\Big|_{-1}^{3}$
$= \frac{3^3}{3}+3-\left(\frac{-1}{3}-1\right)$
$= 12+\frac{4}{3}$
$= \boxed{\frac{40}{3}}$ $\Rightarrow$ $\boxed{\text{The above was an underestimate!}}$

3. (5 points) The half-life of a radioactive substance is 1200 years. Find and simplify $P(t)$, the amount of substance remaining after t years.

$r = \frac{\ln 2}{1200}$

$\Rightarrow \boxed{P(t) = P_0 e^{-\frac{\ln 2}{1200}t}}$

or

$\boxed{P(t) = P_0 \cdot 2^{-t/1200}}$

4. (5 points) Complete the following formulas.

(a) $\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$

(b) $\int 1/x\ dx = \ln|x| + C,\ x \neq 0.$

(c) $\int e^{kx}\ dx = \frac{1}{k}e^{kx} + C,\ k \neq 0.$

Always believe in yourself, no matter what.