Math 205 Test 1

June 22, 2016

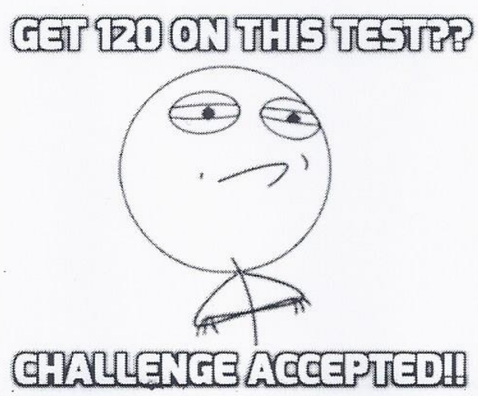
Name: JHEVON SMITH

Note that both sides of each page may have printed material.

Instructions:

1. Read the instructions.
2. Panic!!! Kidding, don't panic! I repeat, do NOT panic!
3. Complete all problems in the actual test. Bonus problems are, of course, optional. And they will only be counted if all other problems are attempted.
4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers.
5. Write neatly so that I am able to follow your sequence of steps and box your answers.
6. Read through the exam and complete the problems that are easy (for you) first!
7. Scientific calculators are needed, but you are NOT allowed to use notes, or other aids—including, but not limited to, divine intervention/inspiration, the internet, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.
8. In fact, **cell phones should be out of sight!**
9. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly...
10. Other than that, have fun and good luck!

May the force be with you. But you can't ask it to help you with your test.



1. (5 points each) Simplify the following:

(a) $\dfrac{5(x^2+2)^4(2x)(x^3-7)^6-(x^2+2)^5(6)(x^3-7)^5(3x^2)}{(x^3-7)^{12}}$

$$= \frac{2x(x^2+2)^4(\cancel{x^3-7})^5[5(x^3-7)-(x^2+2)(3)(3x)]}{(x^3-7)^{\cancel{12}\,7}}$$

$$= \frac{2x(x^2+2)^4[5x^3-35-9x^3-18x]}{(x^3-7)^7}$$

$$= \boxed{\frac{2x(x^2+2)^4(-4x^3-18x-35)}{(x^3-7)^7}} \quad \text{OR} \quad \boxed{\frac{-2x(x^2+2)^4(4x^3+18x+35)}{(x^3-7)^7}}$$

(b) $\ln\sqrt{\dfrac{2e^{5x}\sqrt{x}}{x^3(x+1)^2}} = \frac{1}{2}\ln\left(\dfrac{2e^{5x}x^{1/2}}{x^3(x+1)^2}\right)$

$$= \frac{1}{2}\left[\ln 2 + \cancel{\ln} e^{5x} + \frac{1}{2}\ln x - 3\ln x - 2\ln(x+1)\right]$$

$$= \boxed{\frac{1}{2}\left(\ln 2 + 5x - \frac{5}{2}\ln x - 2\ln(x+1)\right)}$$

(c) $e^{2\ln(5x)-3+\left(\frac{1}{2}\right)\ln y}$

$$= \cancel{e}^{\cancel{\ln}(5x)^2} \cdot e^{-3} \cdot \cancel{e}^{\cancel{\ln} y^{1/2}}$$

$$= \boxed{\frac{(5x)^2 y^{1/2}}{e^3}}$$

(d) $\dfrac{\frac{1}{(x+h)^3}-\frac{1}{x^3}}{h} \cdot \dfrac{x^3(x+h)^3}{x^3(x+h)^3} = \dfrac{x^3-(x+h)^3}{h\,x^3(x+h)^3}$

$$= \frac{\cancel{x^3}-\cancel{x^3}-3x^2h-3xh^2-h^3}{h\,x^3(x+h)^3}$$

$$= \frac{-\cancel{h}(3x^2+3xh+h^2)}{\cancel{h}x^3(x+h)^3}$$

$$= \boxed{\frac{-(3x^2+3xh+h^2)}{x^3(x+h)^3}}$$

2. (a) (4 points each) Find the domains of the following functions:

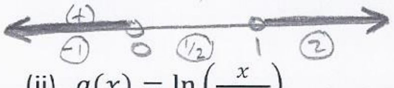
(i) $f(x) = \frac{3x+4}{\sqrt{x^2-x}}$ We need $x^2-x \geq 0$ and $x^2-x \neq 0 \Rightarrow x^2-x > 0$

If $x^2-x=0$

$\Rightarrow x(x-1)=0$

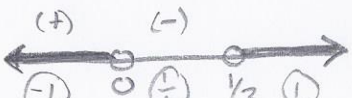
$\Rightarrow x=0, x=1$

$\Rightarrow \boxed{dom(f) = (-\infty,0)\cup(1,\infty)}$



(ii) $g(x) = \ln\left(\frac{x}{3x-1}\right)$

We need $\frac{x}{3x-1} > 0$. Set $x=0$, $3x-1=0 \Rightarrow x=1/3$:



$\Rightarrow \boxed{dom(g) = (-\infty,0)\cup(1/3,\infty)}$

(b) (4 points) Find the equation of the line that passes through $(1,-2)$ that is perpendicular to $3x-2y=4$. $\Rightarrow y = \frac{3}{2}x-2 \Rightarrow m_{new} = -2/3$

$\Rightarrow \boxed{y+2 = -\frac{2}{3}(x-1)}$ or $\boxed{y = -\frac{2}{3}x - 4/3}$

dom: $(-\infty,-2]\cup[2,\infty)$ dom: $(-\infty,\infty)$

(c) (4 points each) Find and simplify the indicated compositions, given $f(x) = \sqrt{2x^2+4}$ and $g(x) = \sqrt{x^2-4}$, and state the domains of each composite function:

(i) $f \circ g = f(g(x))$

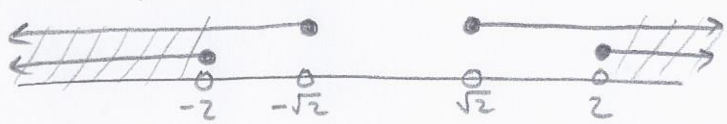
$= \sqrt{2(\sqrt{x^2-4})^2+4}$

$= \sqrt{2x^2-8+4}$

$\boxed{= \sqrt{2(x^2-2)}}$

Note: $dom(g) = (-\infty,-2]\cup[2,\infty)$

$dom(\sqrt{2(x^2-2)}) = (-\infty,-\sqrt{2}]\cup[\sqrt{2},\infty)$



Domain of $f \circ g$: $\boxed{(-\infty,-2]\cup[2,\infty)}$

(ii) $g \circ f = g(f(x))$

$= \sqrt{(\sqrt{2x^2+4})^2-4}$

$= \sqrt{2x^2+4-4}$

$\boxed{= \sqrt{2x^2}}$

Note: $dom(f) = (-\infty,\infty)$

$dom(\sqrt{2x^2}) = (-\infty,\infty)$

Domain of $g \circ f$: $\boxed{(-\infty,\infty)}$

3. (a) (4 points) Let $f(x)$ be a function, state its difference quotient.

$$\boxed{\frac{f(x+h)-f(x)}{h}} \quad \text{or} \quad \boxed{\frac{f(b)-f(a)}{b-a}}$$

(b) (8 points each) Find and simplify the difference quotient of the following functions:

(i) $f(x) = 2 - x^2$

Using $\frac{f(x+h)-f(x)}{h}$

$$= \frac{2-(x+h)^2-(2-x^2)}{h}$$

$$= \frac{\cancel{2}-\cancel{x^2}-2xh-h^2-\cancel{2}+\cancel{x^2}}{h}$$

$$= \frac{\cancel{h}(-2x-h)}{\cancel{h}}$$

$$= \boxed{-2x-h}$$

OR

Using $\frac{f(b)-f(a)}{b-a}$

$$= \frac{2-b^2-(2-a^2)}{b-a}$$

$$= \frac{\cancel{2}-b^2-\cancel{2}+a^2}{b-a}$$

$$= \frac{a^2-b^2}{b-a}$$

$$= \frac{(a-b)(a+b)}{b-a}$$

$$= \boxed{-(a+b)}$$

(ii) $f(x) = 1 - \frac{2}{x}$

Using $\frac{f(x+h)-f(x)}{h}$

$$= \frac{\cancel{1}-\frac{2}{x+h}-(\cancel{1}-\frac{2}{x})}{h}$$

$$= \frac{-\frac{2}{x+h}+\frac{2}{x}}{h} \cdot \frac{x(x+h)}{x(x+h)}$$

$$= \frac{-2x+2(x+h)}{h\,x(x+h)}$$

$$= \frac{-2\cancel{x}+2\cancel{x}+2h}{h\,x(x+h)}$$

$$= \boxed{\frac{2}{x(x+h)}}$$

OR

Using $\frac{f(b)-f(a)}{b-a}$

$$= \frac{\cancel{1}-\frac{2}{b}-(\cancel{1}-\frac{2}{a})}{b-a}$$

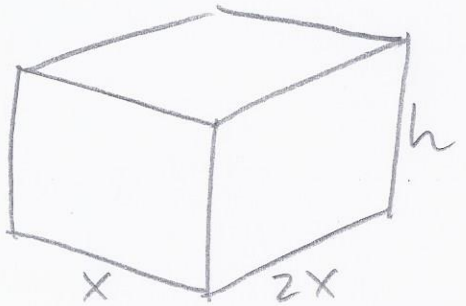
$$= \frac{-\frac{2}{b}+\frac{2}{a}}{b-a} \cdot \frac{ab}{ab}$$

$$= \frac{-2a+2b}{(b-a)ab}$$

$$= \frac{2\cancel{(b-a)}}{\cancel{(b-a)}ab}$$

$$= \boxed{\frac{2}{ab}}$$

4. (a) (5 points) A closed rectangular box with volume 8 ft^3 has length twice the width. Express the height of the box as a function of the width.



$V = 2x^2h$

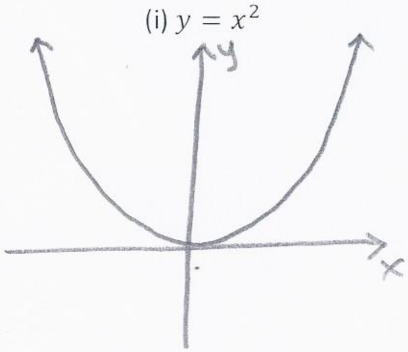
Since $V = 8$

$\Rightarrow 2x^2h = 8$

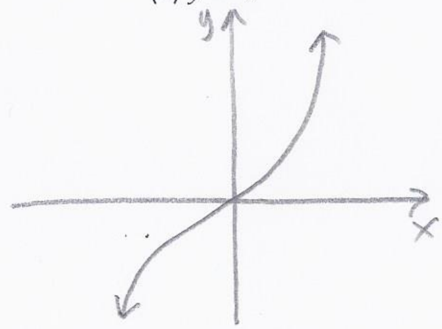
$\Rightarrow \boxed{h = \frac{4}{x^2}}$

(b) (2 points each) Sketch the graphs of the following:

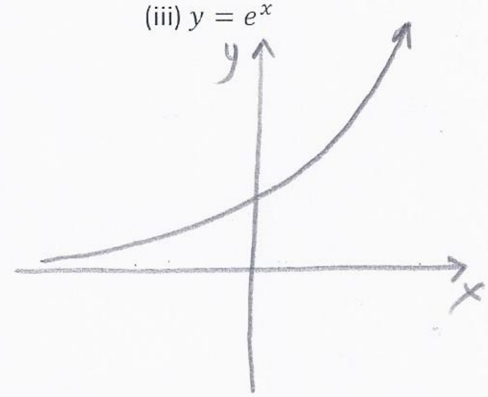
(i) $y = x^2$



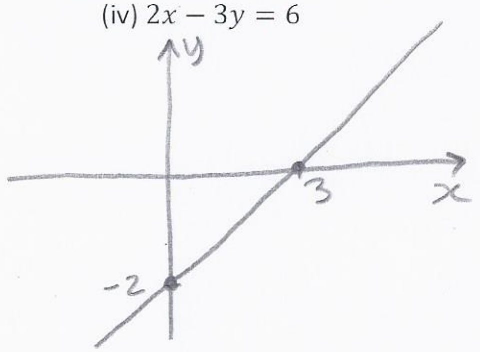
(ii) $y = x^3$



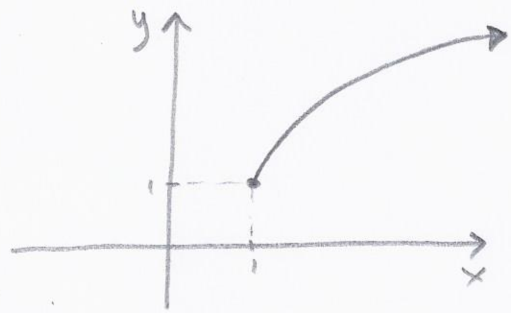
(iii) $y = e^x$



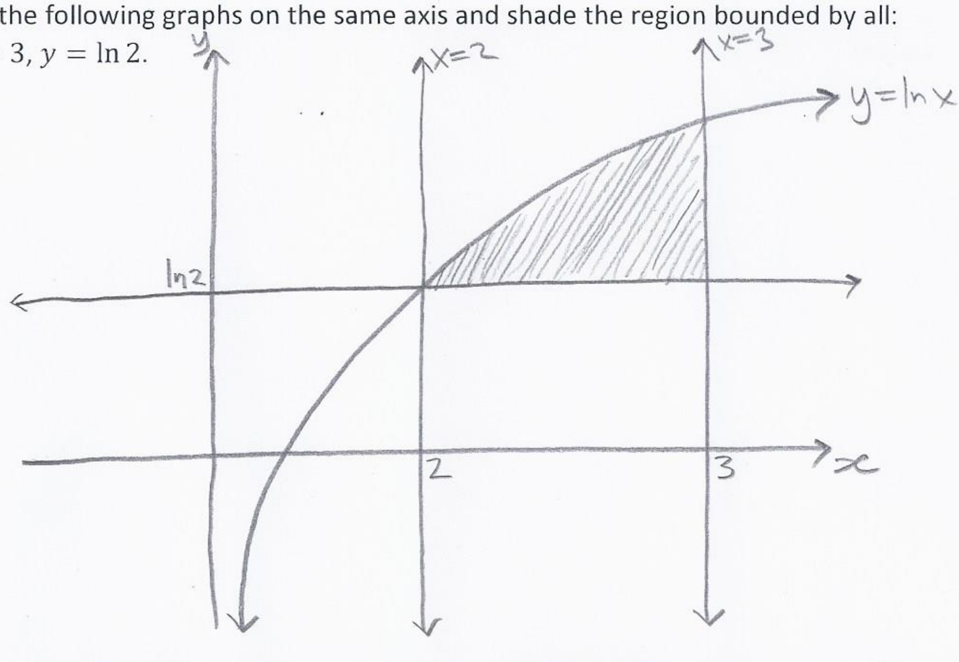
(iv) $2x - 3y = 6$



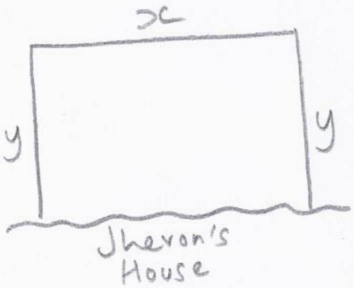
(v) $y = \sqrt{x-1} + 1$



(c) (5 points) Sketch the following graphs on the same axis and shade the region bounded by all: $y = \ln x$, $x = 2$, $x = 3$, $y = \ln 2$.



5. (10 points) Jhevon's llamas are out of control, and he decides to build a rectangular fence enclosure in his backyard to keep them out of trouble. He decides to use 240 feet of fencing to make three sides of the fence, with the back of his house forming the fourth side (because who has time to make a four sided enclosure these days anyway?). Figure out **what dimensions the fence must have** so that his llamas have the most room to frolic. Hint: draw a diagram, and label the sides of the fence x and y. Describe the area as a function of the length of one of the sides, and then find the value so that this area function is as large as possible. Use this to figure out the dimensions.



Fencing = 240 ft.

$\Rightarrow x + 2y = 240$

$\Rightarrow x = 240 - 2y$

$A = xy \longrightarrow$ we want to make this as large as possible.

$= (240-2y)y$ ← It's largest at the vertex.

$= 240y - 2y^2 \longrightarrow$ this is a parabola.

For vertex, $y = \frac{-b}{2a} = -\frac{240}{2(-2)} = 60$ ft

$\Rightarrow x = 240 - 2(60) = 120$ ft.

$\Rightarrow$ $\boxed{\text{Dimensions: } 60\text{ ft} \times 120\text{ ft}; \text{ that is, length} = 120\text{ ft, width} = 60\text{ ft}}$

(b) (5 points each) Solve the following equations:

(i) $e^{2x+3} - 7 = 0$

$\Rightarrow e^{2x+3} = 7$

$\Rightarrow \ln e^{2x+3} = \ln 7$

$\Rightarrow 2x + 3 = \ln 7$

$\Rightarrow \boxed{x = \frac{\ln 7 - 3}{2}}$

(ii) $\ln(5 - 2x) = -3$

$e \qquad e$

$\Rightarrow 5 - 2x = e^{-3}$

$\Rightarrow \boxed{\frac{5 - e^{-3}}{2} = x}$

Bonus Problems (5 points each problem):

1. Compute the following limits:

$(i)\ \lim_{x\to 2}\frac{x^2+x-6}{x^2-x-2}$

$(ii)\ \lim_{x\to 0}\frac{x^2+2}{x^2-1}$

$= \lim_{x\to 2}\frac{(x+3)\cancel{(x-2)}}{(x+1)\cancel{(x-2)}}$

$= \frac{2+3}{2+1}$

$= \boxed{5/3}$

$= \frac{0^2+2}{0-1}$

$= \boxed{-2}$

2. Use the limit definition to find the derivative of $f(x) = \frac{x}{x+1}$.

$f'(x) = \lim_{h\to 0}\frac{\frac{x+h}{x+h+1} - \frac{x}{x+1}}{h}\cdot\frac{(x+1)(x+h+1)}{(x+1)(x+h+1)}$

$= \lim_{h\to 0}\frac{(x+h)(x+1) - x(x+h+1)}{h(x+1)(x+h+1)}$

$= \lim_{h\to 0}\frac{\cancel{x^2}+\cancel{x}+\cancel{xh}+\cancel{h}-\cancel{x^2}-\cancel{xh}-\cancel{x}}{\cancel{h}(x+1)(x+h+1)}$

$= \lim_{h\to 0}\frac{1}{(x+1)(x+h+1)} = \boxed{\frac{1}{(x+1)^2}}$

3. Find derivatives, show your work:

(i) $\frac{d}{dx}\left(\frac{x}{x+1}\right) = \frac{(x+1)-x(1)}{(x+1)^2}$ by quotient rule

$= \boxed{\frac{1}{(x+1)^2}}$

(ii) $\frac{d}{dx}e^{xe^x} = (xe^x)'e^{xe^x}$

$= \boxed{(e^x + xe^x)e^{xe^x}}$

(ii) $\frac{d}{dx}\ln\sqrt{\frac{x+1}{x+2}} = \frac{d}{dx}\frac{1}{2}\left[\ln(x+1) - \ln(x+2)\right]$

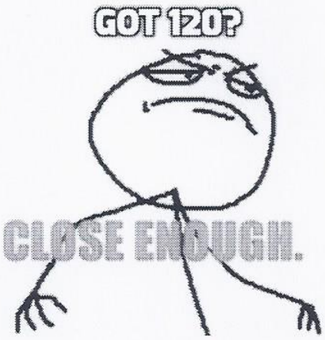
$= \boxed{\frac{1}{2}\left(\frac{1}{x+1} - \frac{1}{x+2}\right)}$

4. Find $\frac{dy}{dx}$ given that $x^2 + y^7 - 2xy = 3$.

$\Rightarrow 2x + 7y^6y' - 2y - 2xy' = 0$

$\Rightarrow (7y^6 - 2x)y' = 2y - 2x$

$\Rightarrow \boxed{\frac{dy}{dx} = \frac{2y-2x}{7y^6-2x}}$


GOT 120?
CLOSE ENOUGH.