MATH 203 TEST 1B

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Note that both sides of each sheet has printed material

Instructions:

1. Read the instructions.
2. Complete all problems!
3. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answer.
4. Write neatly, so that I am able to follow your sequence of steps, and **box your answers**.
5. Read through the exam and kill all the easy problems (for you) first!
6. No calculators, notes, or other outside aids allowed–including the smart kid that may be sitting beside you, or the friend you were thinking of texting.
7. Use correct notation! Write what you mean! "x^2" and "$x2$" are NOT the same thing, and use the right brackets for vectors (don't forget the commas!), for examples.
8. Other than that, have fun, and good luck!

Remember: math is fun, math is beautiful, this test is NOT hard, there is no spoon.

1. (Each response worth 2 points) If $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$.

   (a) How can you tell if $\vec{a}$ and $\vec{b}$ are:

   (i) orthogonal? $\underline{\vec{a} \cdot \vec{b} = 0}$ (ii) parallel? $\underline{\vec{a} = k\vec{b} \text{ or } \vec{a} \times \vec{b} = \vec{0}}$

   (b) What is the formula for $\vec{a} \cdot \vec{b}$? $\vec{a} \cdot \vec{b} = \underline{a_1 b_1 + a_2 b_2 + a_3 b_3}$

   (c) In terms of θ, the angle between $\vec{a}$ and $\vec{b}$, give formulas for:

   (i) $|\vec{a} \times \vec{b}| = \underline{|\vec{a}||\vec{b}| \sin\theta}$

   (ii) $\vec{a} \cdot \vec{b} = \underline{|\vec{a}||\vec{b}| \cos\theta}$

2. State the general equation for the given curve or surface and give the meanings of the symbols used.

   (a) (4 points) A plane

   $a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

   $\langle a,b,c \rangle$ – normal vector to the plane

   (x_0, y_0, z_0) – a point in the plane.

   (b) (2 points each) A line (all three forms. Name the forms!)

   (i) $\langle x,y,z \rangle = \langle x_0, y_0, z_0 \rangle + t\langle a,b,c \rangle$ – vector form

   (ii) $x = x_0 + at,\ y = y_0 + bt,\ z = z_0 + ct$ – parametric form

   (iii) $\dfrac{x-x_0}{a} = \dfrac{y-y_0}{b} = \dfrac{z-z_0}{c}$ – symmetric form

   (x_0, y_0, z) – a point on the line

   $\langle a,b,c \rangle$ – direction vector of the line.

3. (20 points) Find the equation of the plane that contains the line of intersection of the planes $P_1:\ x+y+z=1$ and $P_2:\ x-2y+3z=1$ and contains the point (1,0,1).

Find the line of intersection:

1 point (in the xy-plane): $z=0$

$\Rightarrow$ $x+y=1$ — 1

$x-2y=1$ — 2

$3y=0$ — 1 − 2

$\boxed{y=0}$

$\Rightarrow \boxed{x=1}$

$\Rightarrow$ point is: $(1,0,0)$

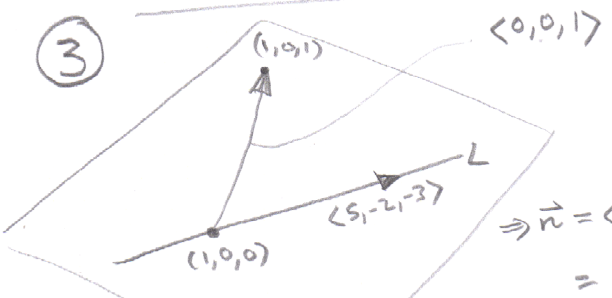
2 direction: $\langle 1,1,1 \rangle \times \langle 1,-2,3 \rangle$

$$= \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix}$$

$= \langle 5,-2,-3 \rangle$

So the line is: $\langle x,y,z \rangle = \langle 1,0,0 \rangle + t\langle 5,-2,-3 \rangle \to L$

3



$\Rightarrow \vec{n} = \langle 0,0,1 \rangle \times \langle 5,-2,-3 \rangle$

$$= \begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ 5 & -2 & -3 \end{vmatrix}$$

$= \langle 2,5,0 \rangle$

4 So the plane is:

$\boxed{2(x-1) + 5y = 0}$

(b) (10 points) Parametrize the line through the origin that is parallel to P_1 and P_2 above. $\langle a,b,c\rangle = \langle 5,-2,-3\rangle$, $(x_0,y_0,z_0) = (0,0,0)$

So the line is: $x = 0+5t$, $y = 0-2t$, $z = 0-3t$

i.e. $\boxed{x = 5t,\ y = -2t,\ z = -3t}$

(c) (10 points) Find the equation, in any form, of the normal line to the plane found in (a) that passes through (1,0,1)

$\langle a,b,c\rangle = \langle 2,5,0\rangle$, $(x_0,y_0,z_0) = (1,0,1)$

$\Rightarrow$ the line is: $\boxed{\langle x,y,z\rangle = \langle 1,0,1\rangle + t\langle 2,5,0\rangle}$

4. (15 points) Find the equation of the tangent line to the curve $\vec{r}(t) = \langle te^t,\ t+\ln t,\ t^3+1\rangle$ at the point $t = 1$

$\vec{r}\,'(t) = \langle e^t + te^t,\ 1+\frac{1}{t},\ 3t^2\rangle$

$\Rightarrow \vec{r}(1) = \langle e, 1, 2\rangle = \langle x_0, y_0, z_0\rangle$

$\Rightarrow \vec{r}\,'(1) = \langle 2e, 2, 3\rangle = \langle a,b,c\rangle.$

$\Rightarrow$ the line is: $\boxed{\langle x,y,z\rangle = \langle e,1,2\rangle + t\langle 2e,2,3\rangle}$

(b) (10 points) Is the line above parallel, perpendicular, or neither to the line found in **3 (b)**? Justify your answer.

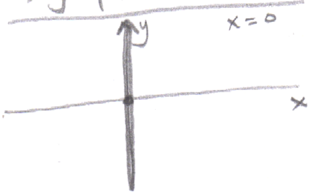
Neither: $\langle 2e,2,3\rangle$ is neither parallel nor perpendicular to $\langle 5,-2,-3\rangle$

5. Sketch the following surfaces. Draw the traces in the coordinate planes as a part of your answer.

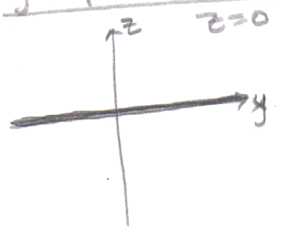
(a) (4 points) $z = x^2$

Parabolic cylinder!

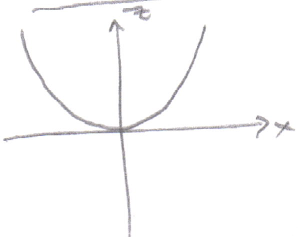
xy-plane: $z=0$



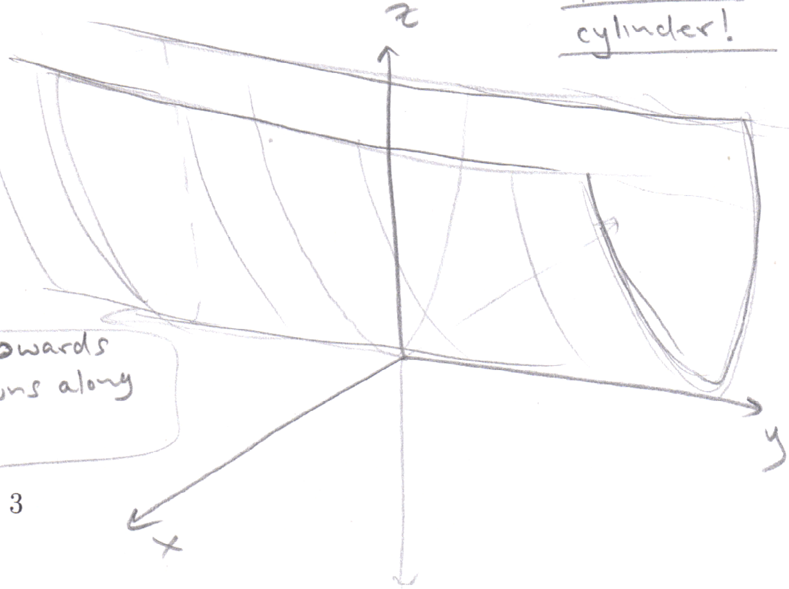
yz-plane: $x=0$



xz-plane: $y=0$



Opens towards the z, runs along the y!

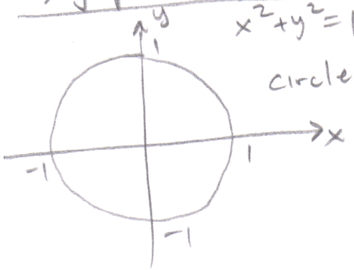


(b) (6 points) $x^2 + y^2 + \frac{z^2}{16} = 1$

xy-plane: $z=0$

$x^2+y^2=1$

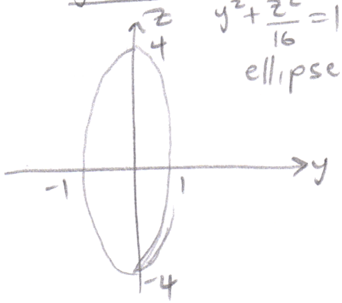
circle!



yz-plane: $x=0$

$y^2+\frac{z^2}{16}=1$

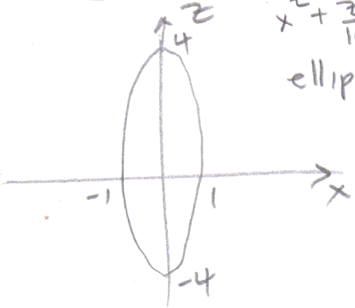
ellipse!



xz-plane: $y=0$

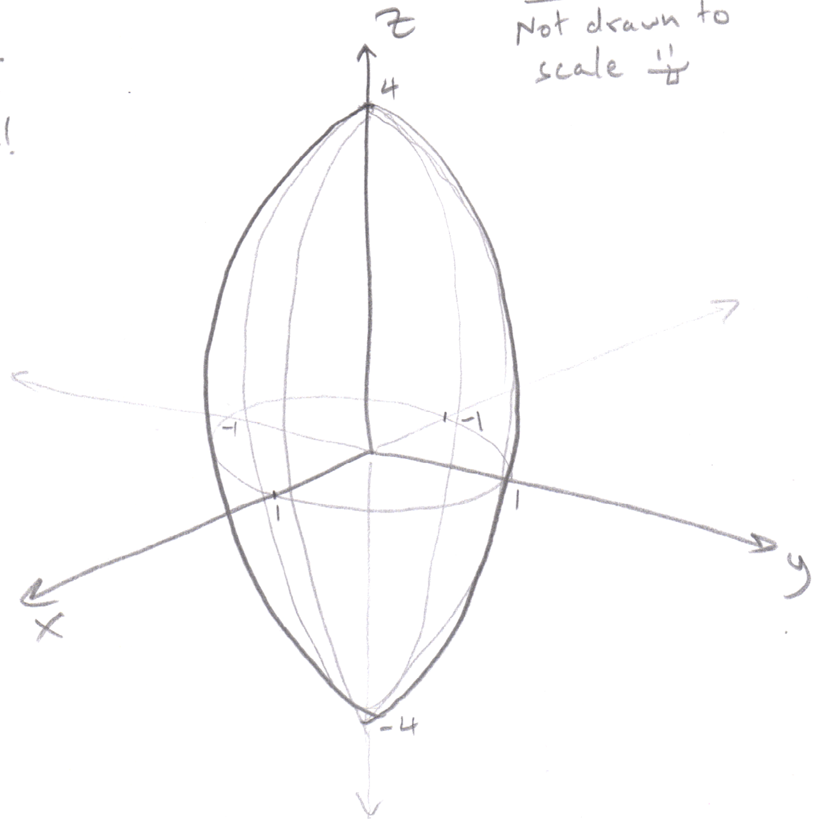
$x^2+\frac{z^2}{16}=1$

ellipse!



Ellipsoid!

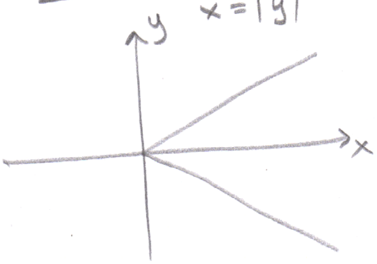
Not drawn to scale :-)



(c) (5 points) $x = \sqrt{y^2 + z^2}$

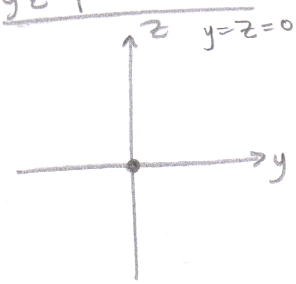
xy-plane: $z=0$

$x=|y|$



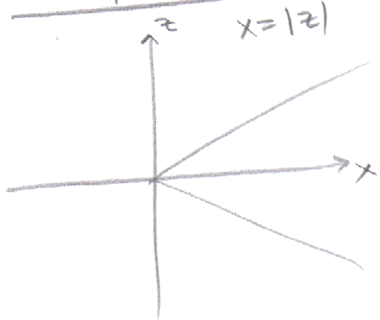
yz-plane: $x=0$

$y=z=0$



xz-plane: $y=0$

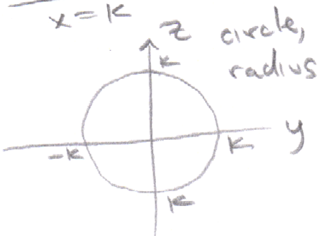
$x=|z|$



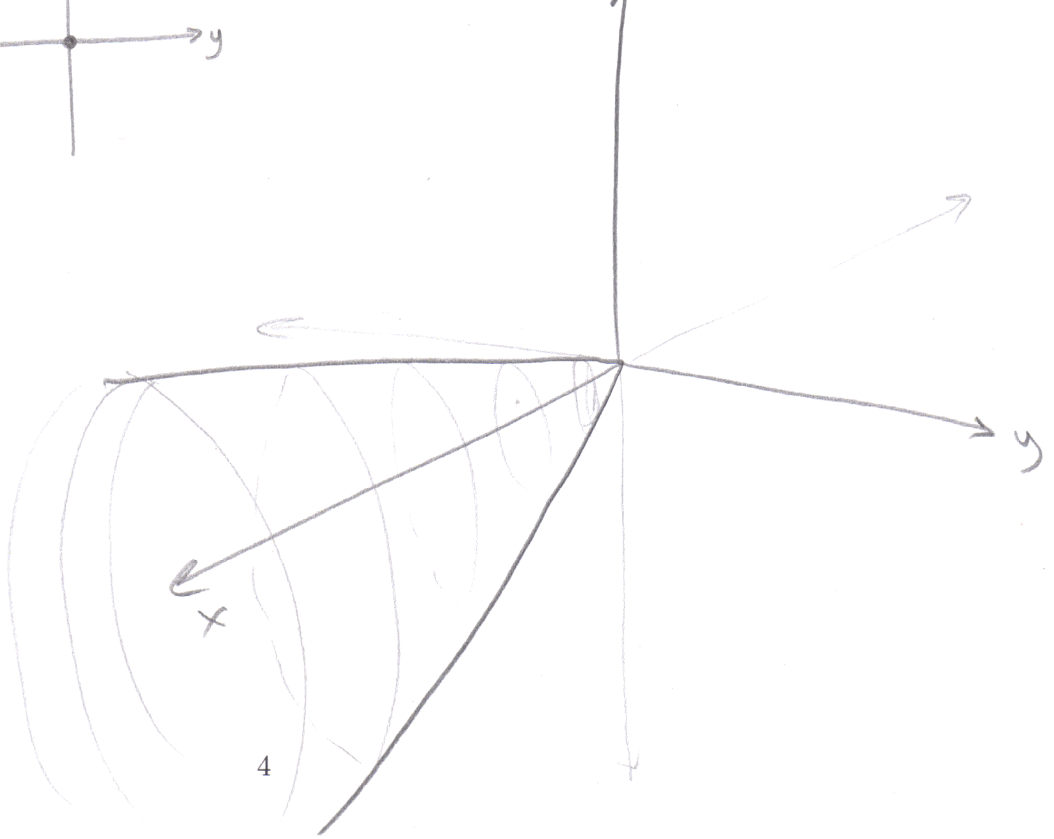
More info:

$x=k$

circle, radius k!



Right-circular cone!



Bonus 1: (10 points) Show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} = L$$

approach (0,0) along $x=0$, we get $L=0$

approach (0,0) along $x=y$, we get $L=\lim\limits_{(x,y)\to(0,0)} \frac{x^2}{2x^2} = \frac{1}{2} \neq 0$

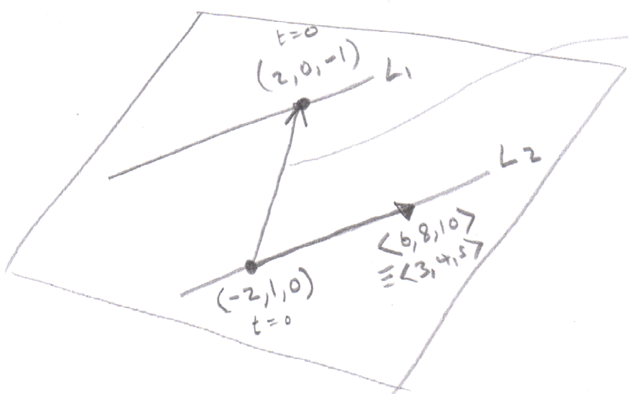
$\therefore$ the limit DNE!

Bonus 2: (10 points) Given the two lines $\underbrace{\frac{x-2}{3} = \frac{y}{4} = \frac{z+1}{5}}_{L_1}$ and $\underbrace{\frac{x+2}{6} = \frac{y-1}{8} = \frac{z}{10}}_{L_2}$

(a) Are they parallel? Explain.

Yes! L_1's direction vector is $\langle 3,4,5\rangle$.
L_2's direction vector is $\langle 6,8,10\rangle$.
Since $\langle 6,8,10\rangle = 2\langle 3,4,5\rangle$, the direction vectors are parallel, and hence the lines are!

(b) Find an equation of the plane containing the two lines.



$\langle 4,-1,-1\rangle$

Hence $\vec{n} = \langle a,b,c\rangle = \begin{vmatrix} i & j & k \\ 3 & 4 & 5 \\ 4 & -1 & -1 \end{vmatrix}$

$= \langle 1, 23, -19\rangle$

take $(x_0,y_0,z_0) = (2,0,-1)$

then the plane is: $\boxed{(x-2)+23y-19(z+1)=0}$

OR

$\boxed{x+23y-19z=21}$