MATH 203 QUIZ 7 - Version A

June 26, 2014

Name: ANSWERS

Instructions: (1) No calculators! (2) Use your own scrap paper. (3) Write your answers in the space provided. Assume all functions are differentiable.

1. For the function $f(x, y, z)$, define $\nabla f = \langle f_x, f_y, f_z \rangle$

2. Suppose $F(x, y, z) = 0$ defines a level surface. Write the formula for the equation of the tangent plane to $F(x, y, z) = 0$ at the point (a, b, c).

$F_x(x-a)+F_y(y-b)+F_z(z-c)=0$ where F_x, F_y, F_z are evaluated at (a,b,c)

3. Using a dot product, define $D_{\mathbf{u}}f = \nabla f \cdot \vec{u}$

4. Let $f(x, y, z) = 3x^2 + 4y^3 - 2xy + 4z$. (a) Find the directional derivative of f at the point $(1, -1, 2)$ in the direction of the point $(2, 0, 4)$.

(i) $\vec{u} = \frac{\langle 1,1,2 \rangle}{\sqrt{6}}$ (ii) $D_{\vec{u}}f = 26/\sqrt{6}$

(b) What is the maximum rate of change at the point $(1, -1, 2)$? $|\nabla f| = 6\sqrt{5}$

(c) Give a unit vector for the direction of the max rate of change. $\left\langle \frac{8}{6\sqrt{5}}, \frac{10}{6\sqrt{5}}, \frac{4}{6\sqrt{5}} \right\rangle = \left\langle \frac{4}{3\sqrt{5}}, \frac{5}{3\sqrt{5}}, \frac{2}{3\sqrt{5}} \right\rangle$

5. (a) Define "D", the formula used to classify the critical points of a function $f(x, y)$.

$D = f_{xx}f_{yy} - (f_{xy})^2$

(b) Find and classify the critical points of $f(x, y) = xy(1 - x - y)$. (No credit for classification if the wrong critical point is given. So solve for them carefully!)

Critical point 1: $(0, 0)$ Classification saddle point

Critical point 2: $(0, 1)$ Classification saddle point

Critical point 3: $(1, 0)$ Classification saddle point

Critical point 4: $\left(\frac{1}{3}, \frac{1}{3}\right)$ Classification maximum point

Bonus 1: Set up an integral to compute the volume of the solid in the first octant bounded by the cylinder $z = 16 - x^2$ and the plane $y = 5$.

Integral set-up: $\int_0^5\int_0^4 16-x^2\,dx\,dy$ or $\int_0^4\int_0^5 16-x^2\,dy\,dx$ Volume $\frac{640}{3}$

Bonus 2: Evaluate $\int_0^1 \int_x^1 e^{x/y}\, dy\, dx = \frac{e-1}{2}$. Hint: reverse the order of integration.