

MATH 203 QUIZ 6 - Version B

June 23, 2014

Name: ANSWERS

**Instructions:** (1) No calculators! (2) Use your own scrap paper. (3) Write your answers in the space provided. Assume all functions are differentiable.

1. Suppose  $w = f(x, y, z)$  and  $x = x(q, r, s)$ ,  $y = y(q, r, s)$  and  $z = z(q, r, s)$ , write down a formula for

$$\frac{\partial w}{\partial q} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial q} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial q} + \frac{\partial f}{\partial z} \frac{\partial z}{\partial q} \quad \text{or} \quad f_x x_q + f_y y_q + f_z z_q \quad (\text{may use } w \text{ instead of } f)$$

2. Find the indicated derivative for the given function.

(a)  $z = x \ln(3x + y)$ ,  $x = \sin t$ ,  $y = \cos t$ .  $\frac{\partial z}{\partial t} = \left( \ln(3x+y) + \frac{3x}{3x+y} \right) \cos t - \frac{x}{3x+y} \sin t$

(b)  $z = 2x/y$ ,  $x = se^{-t}$ ,  $y = 1 - se^t$ .  $\frac{\partial z}{\partial s} = \frac{2}{y} e^{-t} + \frac{2x}{y^2} e^t$

(c) For the above problem, find  $z_t(2, -1) = \underline{2}$

3. Suppose  $W(s, t) = F(u(s, t), v(s, t))$ . Also,  $u(1, 0) = 2$ ,  $u_s(1, 0) = -2$ ,  $u_t(1, 0) = 6$ ,  $v(1, 0) = 3$ ,  $v_s(1, 0) = 5$ ,  $v_t(1, 0) = 4$ ,  $F_u(2, 3) = -1$  and  $F_v(2, 3) = 10$ . Find  $W_t(1, 0)$ .

$W_t(1, 0) = \underline{34}$

4. A function  $z = f(x, y)$  is defined implicitly by  $xyz = \cos(x + y + z) - \ln\left(\frac{xy}{z}\right)$ . What is:

$$\frac{\partial z}{\partial y} = \frac{xz + \sin(x+y+z) + \frac{1}{y}}{xy + \sin(x+y+z) - \frac{1}{z}}$$

**Bonus 1:** Let  $D_{\mathbf{u}}f$  be the directional derivative of a function  $f(x, y)$  in the direction of the unit vector  $\mathbf{u}$ .

(a) Using a dot product, define  $D_{\mathbf{u}}f = \underline{\nabla f \cdot \mathbf{u}}$

(b) Using limits, define  $D_{\mathbf{u}}f = \underline{\lim_{h \rightarrow 0} \frac{f(x+ah, y+bh) - f(x, y)}{h}}$

**Bonus 2:** Suppose  $f = f(x, y)$ . Define  $\nabla f = \underline{\langle f_x, f_y \rangle}$