MATH 203 QUIZ 6 - Version A

June 23, 2014

Name: ANSWERS

Instructions: (1) No calculators! (2) Use your own scrap paper. (3) Write your answers in the space provided. Assume all functions are differentiable.

1. Suppose $w = f(x, y, z)$ and $x = x(q, r, s)$, $y = y(q, r, s)$ and $z = z(q, r, s)$, write down a formula for

$$\frac{\partial w}{\partial r} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial r} \quad \text{or} \quad f_x x_r + f_y y_r + f_z z_r$$

(may use w instead of f)

2. Find the indicated derivative for the given function.

(a) $z = x\ln(x + 2y)$, $x = \sin t$, $y = \cos t$. $\dfrac{\partial z}{\partial t} = \left(\ln(x+2y) + \dfrac{x}{x+2y}\right)\cos t - \dfrac{2x}{x+2y}\sin t$

(b) $z = 2x/y$, $x = se^{-t}$, $y = 1 + se^{t}$. $\dfrac{\partial z}{\partial s} = \dfrac{2e^{-t}}{y} - \dfrac{2x}{y^2}e^{t}$

(c) For the above problem, find $z_t(2, 3) = \frac{2}{3} - \frac{8}{9} = -2/9$

3. Suppose $W(s, t) = F(u(s, t), v(s, t))$. Also, $u(1, 0) = 2$, $u_s(1, 0) = -2$, $u_t(1, 0) = 6$, $v(1, 0) = 3$, $v_s(1, 0) = 5$, $v_t(1, 0) = 4$, $F_u(2, 3) = -1$ and $F_v(2, 3) = 10$. Find $W_s(1, 0)$.

$W_s(1, 0) =$ 52

4. A function $z = f(x, y)$ is defined implicitly by $xyz = \cos(x + y + z) - \ln\left(\frac{xy}{z}\right)$. What is:

$$\frac{\partial z}{\partial x} = -\frac{yz + \sin(x+y+z) + \frac{1}{x}}{xy + \sin(x+y+z) - 1/z}$$

Bonus 1: Suppose $f = f(x, y, z)$. Define $\nabla f = \langle f_x, f_y, f_z \rangle$

Bonus 2: Let $D_{\mathbf{u}}f$ be the directional derivative of a function $f(x, y)$ in the direction of the unit vector $\mathbf{u}$.

(a) Using limits, define $D_{\mathbf{u}}f = \lim_{h \to 0} \dfrac{f(x+ah, y+bh) - f(x,y)}{h}$, where $\vec{u} = \langle a, b \rangle$

(b) Using a dot product, define $D_{\mathbf{u}}f = \nabla f \cdot \vec{u}$