Math 203 Test 1B
October 1, 2015

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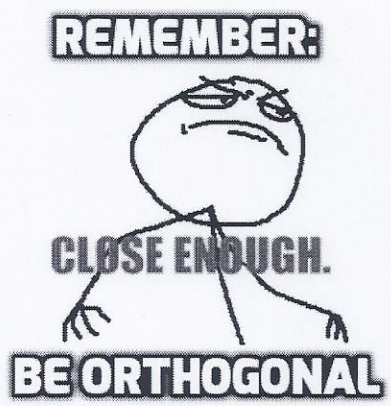
Note that both sides of each page may have printed material.

Instructions:

1. Read the instructions.
2. Panic!!! Kidding, don't panic! I repeat, do NOT panic!
3. Complete all problems in the actual test. Bonus problems are, of course, optional.
4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers (unless it's a case of fill in the blank or state a definition, etc.)
5. Write neatly so that I am able to follow your sequence of steps and box your answers.
6. Read through the exam and complete the problems that are easy (for you) first!
7. No scrap paper, calculators, notes or other outside aids allowed—including divine intervention, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.
8. In fact, **cell phones should be out of sight!**
9. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.
10. Do NOT commit any of the blasphemies or mistakes I mentioned in the syllabus. I will actually mete out punishment in the way I said I would. I wasn't kidding.
11. Other than that, have fun and good luck!



1. (Each response worth 2 points) If $\vec{a} = \langle a_1, a_2, a_3\rangle$ and $\vec{b} = \langle b_1, b_2, b_3\rangle$.

   (a) How can you tell if $\vec{a}$ and $\vec{b}$ are:

   (i) orthogonal? $\vec{a}\cdot\vec{b} = 0$ (ii) parallel? $\vec{a} = k\vec{b}$ or $\vec{a}\times\vec{b} = \vec{0}$

   (b) What is the formula for $\vec{a}\cdot\vec{b}$? $\vec{a}\cdot\vec{b} =$ $a_1b_1 + a_2b_2 + a_3b_3$

   (c) In terms of θ, the angle between $\vec{a}$ and $\vec{b}$, give formulas for:

   (i) $|\vec{a}\times\vec{b}| =$ $|\vec{a}||\vec{b}|\sin\theta$

   (ii) $\vec{a}\cdot\vec{b} =$ $|\vec{a}||\vec{b}|\cos\theta$

2. State the general equation for the given curve or surface and give the meanings of the symbols used.

   (a) (4 points) A plane

   $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

   $\langle a,b,c\rangle$ – normal vector to the plane

   (x_0,y_0,z_0) – a point in the plane.

   (b) (2 points each) A line (all three forms. Name the forms!)

   (i) $\langle x,y,z\rangle = \langle x_0,y_0,z_0\rangle + t\langle a,b,c\rangle$ – vector form

   (ii) $x = x_0+at,\ y = y_0+bt,\ z = z_0+ct$ – parametric form

   (iii) $\dfrac{x-x_0}{a} = \dfrac{y-y_0}{b} = \dfrac{z-z_0}{c}$ – symmetric form

   (x_0,y_0,z_0) – a point on the line

   $\langle a,b,c\rangle$ – direction vector of the line.

3. (20 points) Find the equation of the plane that contains the line of intersection of the planes $P_1:\ x+y+z=1$ and $P_2:\ x-2y+3z=1$ and contains the point (1,0,1).

Find the line of intersection:

1 point (in the xy-plane): $z=0$

$\Rightarrow$ $x+y=1$ — ①

$x-2y=1$ — ②

$3y=0$ — ①−②

$\boxed{y=0}$

$\Rightarrow \boxed{x=1}$

$\Rightarrow$ point is: $(1,0,0)$

2 direction: $\langle 1,1,1\rangle\times\langle 1,-2,3\rangle$

$$= \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix}$$

$= \langle 5,-2,-3\rangle$

So the line is: $\underbrace{\langle x,y,z\rangle = \langle 1,0,0\rangle + t\langle 5,-2,-3\rangle}_{L}$

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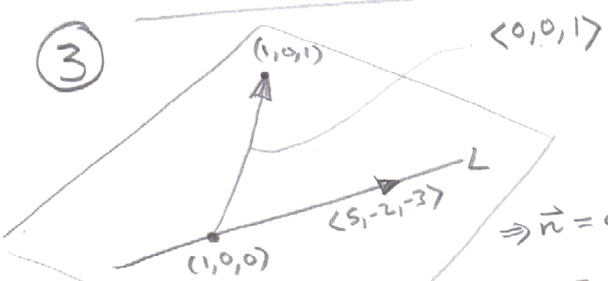


$\Rightarrow \vec{n} = \langle 0,0,1\rangle\times\langle 5,-2,-3\rangle$

$$= \begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ 5 & -2 & -3 \end{vmatrix}$$

$= \langle 2,5,0\rangle$

4 So the plane is:

$\boxed{2(x-1)+5y=0}$

(b) (10 points) Parametrize the line through the origin that is parallel to P_1 and P_2 above. $\langle a,b,c\rangle = \langle 5,-2,-3\rangle$, $(x_0,y_0,z_0)=(0,0,0)$

So the line is: $x=0+5t$, $y=0-2t$, $z=0-3t$

i.e. $\boxed{x=5t,\ y=-2t,\ z=-3t}$

(c) (10 points) Find the equation, in any form, of the normal line to the plane found in (a) that passes through (1,0,1)

$\langle a,b,c\rangle = \langle 2,5,0\rangle$, $(x_0,y_0,z_0)=(1,0,1)$

$\Rightarrow$ the line is: $\boxed{\langle x,y,z\rangle = \langle 1,0,1\rangle + t\langle 2,5,0\rangle}$

4. (15 points) Find the equation of the tangent line to the curve $\vec{r}(t) = \langle te^t,\ t+\ln t,\ t^3+1\rangle$ at the point $t=1$

$\vec{r}\,'(t) = \langle e^t + te^t,\ 1+\frac{1}{t},\ 3t^2\rangle$

$\Rightarrow \vec{r}(1) = \langle e,1,2\rangle = \langle x_0,y_0,z_0\rangle$

$\Rightarrow \vec{r}\,'(1) = \langle 2e,2,3\rangle = \langle a,b,c\rangle.$

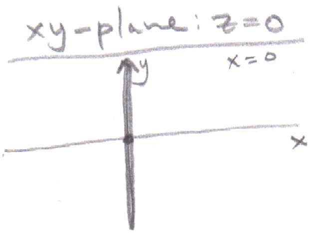
$\Rightarrow$ the line is: $\boxed{\langle x,y,z\rangle = \langle e,1,2\rangle + t\langle 2e,2,3\rangle}$

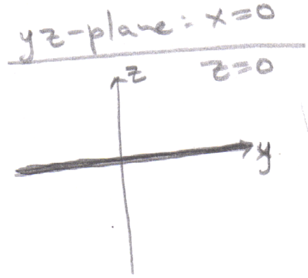
(b) (10 points) Is the line above parallel, perpendicular, or neither to the line found in **3 (b)**? Justify your answer.

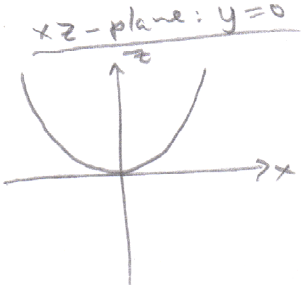
Neither: $\langle 2e,2,3\rangle$ is neither parallel nor perpendicular to $\langle 5,-2,-3\rangle$

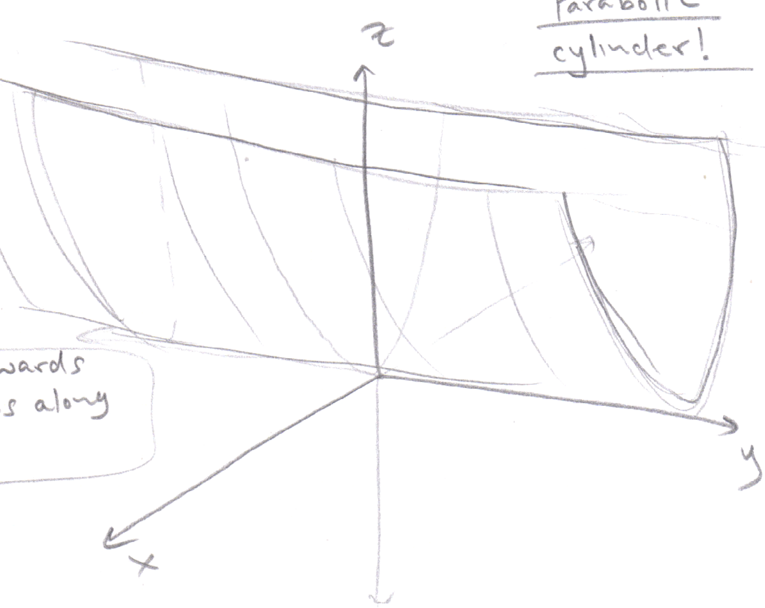
5. Sketch the following surfaces. Draw the traces in the coordinate planes as a part of your answer.

(a) (4 points) $z = x^2$



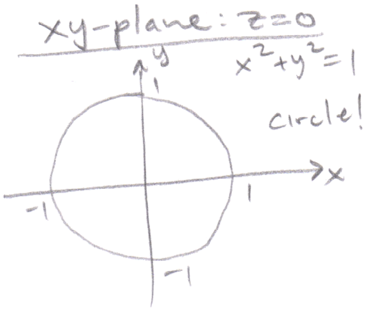


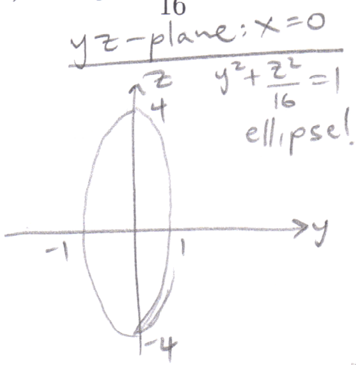


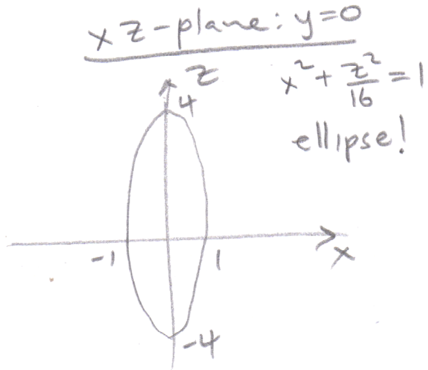


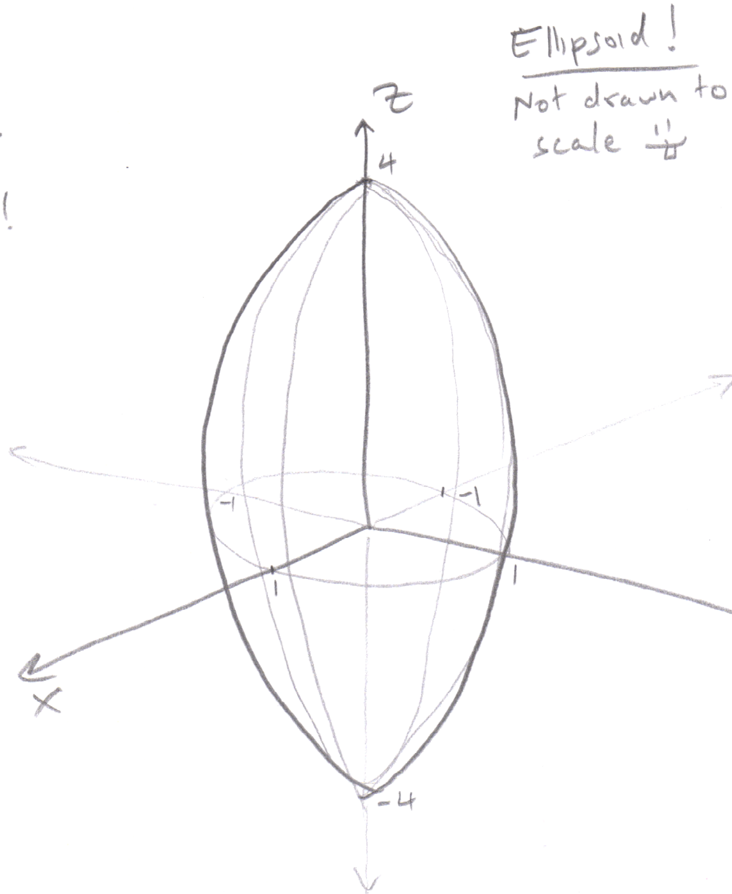
Opens towards the z, runs along the y!

(b) (6 points) $x^2 + y^2 + \frac{z^2}{16} = 1$

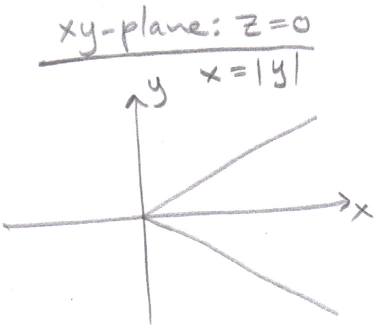


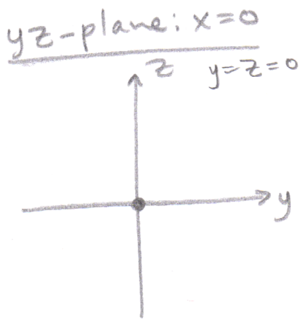


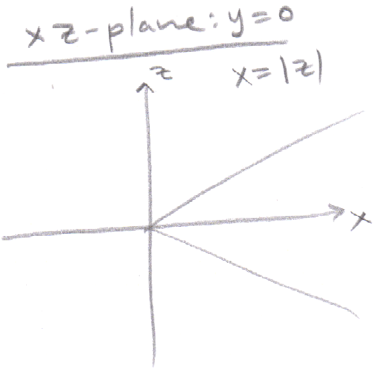


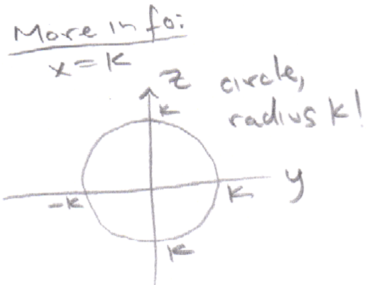


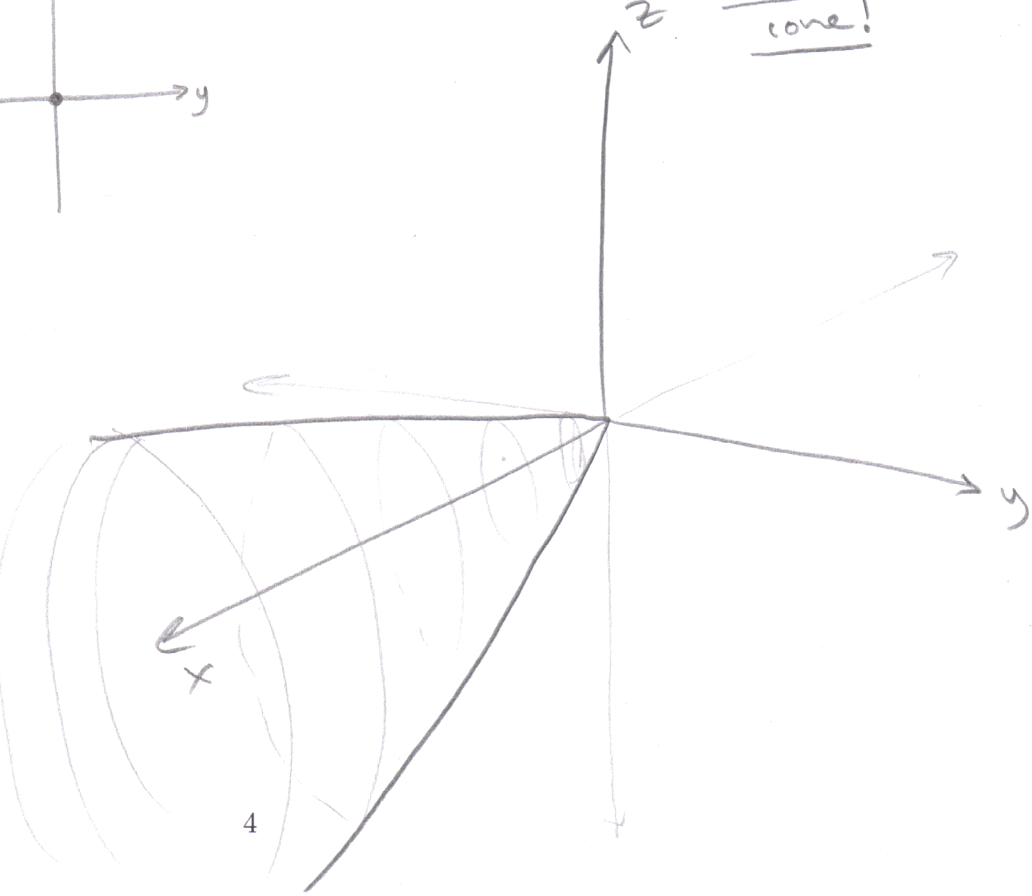
(c) (5 points) $x = \sqrt{y^2 + z^2}$











Bonus 1: (2 points) Show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} = L$$

along $x=0$ (or $y=0$), we get $L=0$

along $x=y$, we get $L = 1/2 \neq 0$

$\Rightarrow$ $\boxed{\text{Limit DNE!}}$

Bonus 2: (8 points) A function $f(x,y)$ is defined as $f(x,y) = \dfrac{x^2y}{\sqrt{x^2+y^2}}$ if $(x,y) \neq (0,0)$ and $f(x,y) = 0$ if $(x,y) = (0,0)$. Show that $f(x,y)$ is continuous everywhere. You must prove your claim.

Clearly $f(x,y)$ is continuous for all $(x,y) \neq (0,0)$.

At $(x,y) = (0,0)$: Note that

$$|f - 0| = \left|\frac{x^2y}{\sqrt{x^2+y^2}}\right| \leq \left|\frac{x^2y}{\sqrt{x^2}}\right| = |xy| \to 0 \text{ as } (x,y)\to(0,0).$$

So that $\lim\limits_{(x,y)\to(0,0)} f(x,y) = 0 = f(0,0)$

$\Rightarrow$ $f(x,y)$ is continuous at $(0,0)$.

$\Rightarrow$ $\boxed{f(x,y) \text{ is continuous everywhere!}}$

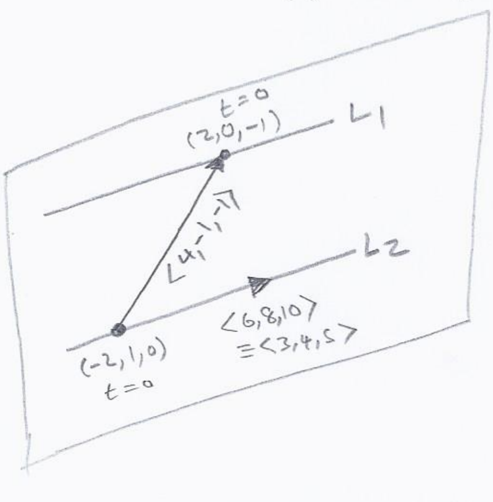
Bonus 3: (5 points) Given the two lines $\dfrac{x-2}{3} = \dfrac{y}{4} = \dfrac{z+1}{5}$ (L_1) and $\dfrac{x+2}{6} = \dfrac{y-1}{8} = \dfrac{z}{10}$ (L_2)

(a) Are they parallel? Explain.

Direction vector for $L_1 = \langle 3,4,5 \rangle = \frac{1}{2}\langle 6,8,10\rangle$.

So the direction vectors are parallel $\Rightarrow$ $\boxed{\text{the lines are parallel}}$

(b) Find an equation of the plane containing the two lines.



$$\Rightarrow \vec{n} = \begin{vmatrix} i & j & k \\ 3 & 4 & 5 \\ 4 & -1 & -1 \end{vmatrix} = \langle 1, 23, -19 \rangle$$

take $(x_0, y_0, z_0) = (2, 0, -1)$

then the plane is

$$\boxed{(x-2) + 23y - 19(z+1) = 0}$$

or

$$\boxed{x + 23y - 19z = 21}$$