Math 203 Test 3A
December 1, 2015

Name: JHEVON SMITH

Note that both sides of each page may have printed material.

Instructions:

1. Read the instructions.

2. Panic!!! Kidding, don't panic! I repeat, do NOT panic!

3. Complete all problems in the actual test. Bonus problems are, of course, optional.

4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers (unless it's a case of fill in the blank or state a definition, etc.)

5. Write neatly so that I am able to follow your sequence of steps and box your answers.

6. Read through the exam and complete the problems that are easy (for you) first!

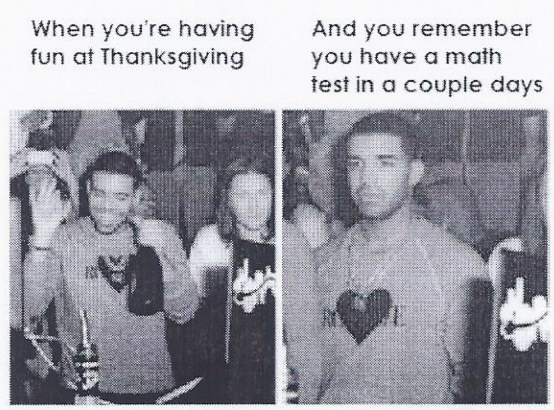
7. No scrap paper, calculators, notes or other outside aids allowed—including divine intervention, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.

8. In fact, **cell phones should be out of sight!**

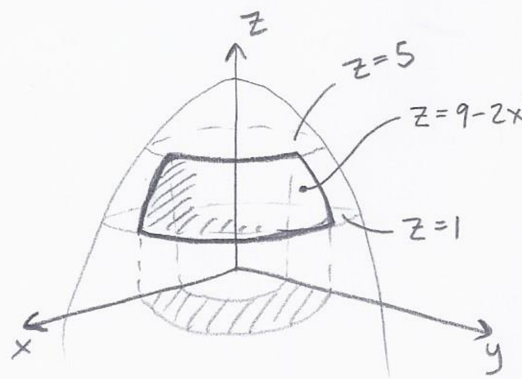
9. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.

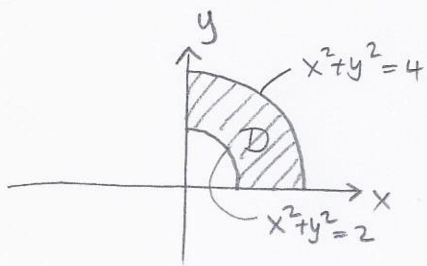
10. Do NOT commit any of the blasphemies or mistakes I mentioned in the syllabus. I will actually mete out punishment in the way I said I would. I wasn't kidding.

11. Other than that, have fun and good luck!



1. (20 points) Let S be the part of the surface $z = 9 - 2x^2 - 2y^2$ between the planes $z = 1$ and $z = 5$ in the first octant. Compute the surface area of S. Include a sketch in your answer.





$z = f = 9 - 2x^2 - 2y^2$

$\Rightarrow f_x = -4x, \ f_y = -4y$

<u>when $z=1$</u>

$1 = 9 - 2x^2 - 2y^2$

$\Rightarrow x^2 + y^2 = 4$

$\Rightarrow r = 2$

<u>when $z=5$</u>

$5 = 9 - 2x^2 - 2y^2$

$\Rightarrow x^2 + y^2 = 2$

$\Rightarrow r = \sqrt{2}$

(5 points for the correct sketch)

$$A = \iint_D \sqrt{1 + f_x^2 + f_y^2} \, dA$$

$$= \iint_D \sqrt{1 + 16x^2 + 16y^2} \, dA$$

$$= \int_0^{\frac{\pi}{2}} \int_{\sqrt{2}}^{2} \sqrt{1 + 16r^2} \, r \, dr \, d\theta$$

(10 pts for correct setup)

(Let $u = 1 + 16r^2$)

$$= \frac{1}{32} \int_0^{\frac{\pi}{2}} \int_{33}^{65} u^{1/2} \, du \, d\theta$$

$$= \frac{1}{32} \cdot \frac{2}{3} \cdot \frac{\pi}{2} \cdot u^{3/2} \Big|_{33}^{65}$$

(4 pts for evaluation)

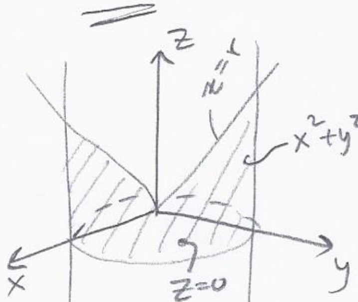
$$= \boxed{\frac{\pi}{96}\left(65^{3/2} - 33^{3/2}\right)}$$

(1 pt for correct final answer)

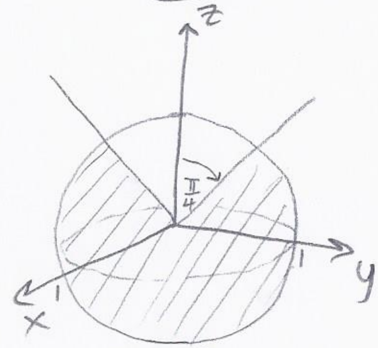
2. Let T_1 be the region below the cone $z = \sqrt{x^2+y^2}$, inside the cylinder $x^2+y^2 = 1$, above the xy-plane. Let T_2 be the region inside the sphere $x^2+y^2+z^2 = 1$, but below the cone $z = \sqrt{x^2+y^2}$.

(a) (10 points) Set up $\iiint_{T_1} (x^2+y^2+z^2)\, dV$ and $\iiint_{T_2} (x^2+y^2+z^2)\, dV$ using coordinate systems of your choice. Justify your answer!

T_1



T_2



$$\iiint_{T_1} = \int_0^{2\pi}\int_0^1\int_0^r (r^2+z^2)\, r\, dz\, dr\, d\theta$$

switch to cylindrical coordinates

(2 pts for diagram or other justification, 3 pts for correct integral set up)

$$\iiint_{T_2} = \int_0^{2\pi}\int_{\frac{\pi}{4}}^{\pi}\int_0^1 \rho^4 \sin\phi\, d\rho\, d\phi\, d\theta$$

Switch to spherical coordinates.

(2 pts for diagram or other justification, 3 pts for correct integral set up)

(b) (20 points) Evaluate either of the integrals set up in 2(a) above.

T_1

$$\int_0^{2\pi}\int_0^1\int_0^r (r^2+z^2)\, r\, dz\, dr\, d\theta$$

$$= 2\pi \int_0^1 r^3 z + r\frac{z^3}{3}\Big|_0^r\, dr$$

$$= 2\pi \int_0^1 r^4 + \frac{r^4}{3}\, dr$$

$$= \frac{8\pi}{3}\int_0^1 r^4\, dr$$

$$= \frac{8\pi}{15} r^5\Big|_0^1$$

$$= \boxed{\frac{8\pi}{15}}$$

(2pts for correct final answer, 18 pts for evaluation)

OR

T_2

$$\int_0^{2\pi}\int_{\frac{\pi}{4}}^{\pi}\int_0^1 \rho^4 \sin\phi\, d\rho\, d\phi\, d\theta$$

$$= 2\pi \int_{\frac{\pi}{4}}^{\pi} \frac{\rho^5}{5}\sin\phi\Big|_0^1\, d\phi$$

$$= \frac{2\pi}{5}\int_{\frac{\pi}{4}}^{\pi} \sin\phi\, d\phi$$

$$= -\frac{2\pi}{5}\cos\phi\Big|_{\pi/4}^{\pi}$$

$$= -\frac{2\pi}{5}\left(-1-\frac{\sqrt{2}}{2}\right)$$

$$= \boxed{\frac{2+\sqrt{2}}{5}\pi}$$

(2 pts for correct final answer, 18 pts for evaluation).

3. (10 points) Write down the formulas for the mass and center of mass of a lamina whose density at the point (x, y) is given by $\rho(x, y)$.

$$\boxed{m = \iint_D \rho(x,y)\, dA}$$

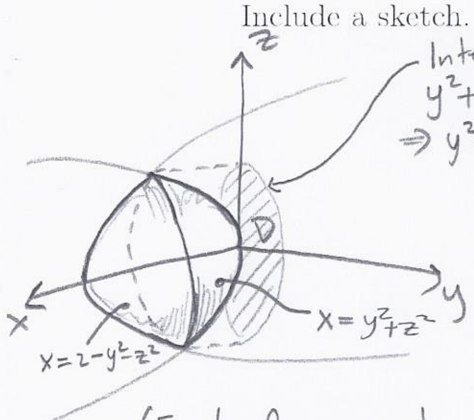
(4 pts)
all or nothing

$$\left.\begin{aligned} \bar{x} &= \frac{1}{m}\iint_D x\,\rho(x,y)\, dA \\ \bar{y} &= \frac{1}{m}\iint_D y\,\rho(x,y)\, dA \end{aligned}\right\} \boxed{C.O.M = (\bar{x}, \bar{y})}$$

(2 pts)
all or nothing

(4 pts)
all or nothing

4. (20 points) Find the volume of the region bounded by $x = y^2 + z^2$ and $x = 2 - y^2 - z^2$. Include a sketch.



Intersection...
$y^2+z^2 = 2-y^2-z^2$
$\Rightarrow y^2+z^2 = 1$

(5 pts for correct sketch)

Now, $V = \iiint_E 1\, dV$ or $\iint_D \text{top-bottom}\, dA$

(10 pts for correct setup)

$$= \int_0^{2\pi}\int_0^1\int_{r^2}^{2-r^2} r\,dx\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^1 [(2-r^2)-r^2]\,r\,dr\,d\theta$$

$$= 2\pi\left[r^2 - \frac{1}{2}r^4\right]\Big|_0^1$$

(4 pts to evaluate)

$$= 2\pi\left[\frac{1}{2}\right]$$

$$\boxed{= \pi}$$

(1 pt for correct final answer)

(b) (20 points) For the solid region described in 4(a) above, find its centroid, assuming it has constant uniform density.

Assume density $= K$. Note that by above, $m = K\pi$

By symmetry, $\bar{y} = \bar{z} = 0$.

$$\bar{x} = \frac{1}{m}\int_0^{2\pi}\int_0^1\int_{r^2}^{2-r^2} Kx\, r\,dx\,dr\,d\theta$$

$$= \frac{1}{\pi}\cdot 2\pi\int_0^1 \frac{x^2}{2}\,r\Big|_{r^2}^{2-r^2} dr$$

$$= 2\int_0^1 \frac{(2-r^2)^2}{2}r - \frac{r^5}{2}\,dr$$

$$= 2\left[-\frac{(2-r^2)^3}{12} - \frac{r^6}{12}\right]\Big|_0^1$$

$$= 2\left[-\frac{1}{12} - \frac{1}{12} + \frac{8}{12}\right]$$

$$= 1$$

$\Rightarrow$ $\boxed{\text{Centroid is } (1, 0, 0)}$

(1 pt for noting density is a constant
2 pts for figuring out m
2 pts for figuring out $\bar{y}$ and $\bar{z}$
10 pts for $\bar{x}$ integral set up
3 pts for integral evaluation
2 pts for stating the centroid).

Bonus 1: (10 points) For each of the series below, state whether the series is convergent (in any sense). Justify your answer.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{2n-5}{n^3-4n+3}$ (b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

Let $a_n = \dfrac{2n-5}{n^3-4n+3}$, $b_n = \dfrac{2n}{n^3} = \dfrac{2}{n^2}$

$\Rightarrow \displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = 1 > 0$

$\Rightarrow \sum a_n$ and $\sum b_n$ do the same thing.

Since $\sum b_n$ converges (it's a convergent p-series)

$\sum a_n$ also converges.

$\Rightarrow \boxed{\displaystyle\sum_{2}^{\infty} \frac{2n-5}{n^3-4n+3} \text{ converges!}}$

by the limit comparison test

(2 pts for correct conclusion
5 pts for knowing what tests may be used
3 pts for proper execution)

Let $a_n = \dfrac{1}{n\sqrt{\ln n}}$.

Since $a_n \geq 0$, a_n is decreasing and a_n is continuous for $n \geq 2$, we can use the integral test.

$$\int_2^{\infty} a_x\,dx = \int_2^{\infty} \frac{1}{x\sqrt{\ln x}}\,dx$$

$$= 2\sqrt{\ln x}\,\Big|_2^{\infty}$$

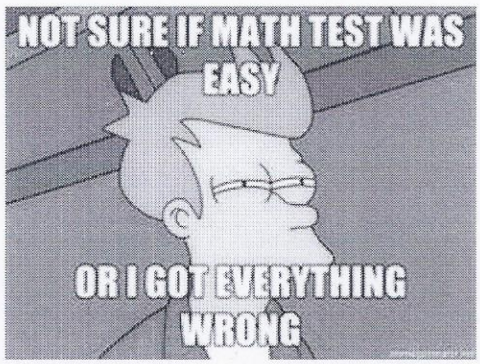
$$= \infty$$

$\Rightarrow \boxed{\displaystyle\sum_{2}^{\infty} \frac{1}{n\sqrt{\ln n}} \text{ diverges}}$ by the integral test

Bonus 2: (5 points) Write down the Maclaurin series for $f(x) = \sin x$.

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

(all or nothing)


NOT SURE IF MATH TEST WAS EASY
OR I GOT EVERYTHING WRONG