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Note that both sides of each page may have printed material.

Instructions:

1. Read the instructions.
2. Panic!!! Kidding, don't panic! I repeat, do NOT panic!
3. Complete all problems in the actual test. Bonus problems are, of course, optional.
4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers (unless it's a case of fill in the blank or state a definition, etc.)
5. Write neatly so that I am able to follow your sequence of steps and box your answers.
6. Read through the exam and complete the problems that are easy (for you) first!
7. No scrap paper, calculators, notes or other outside aids allowed—including divine intervention, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.
8. In fact, **cell phones should be out of sight!**
9. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.
10. Do NOT commit any of the blasphemies or mistakes I mentioned in the syllabus. I will actually mete out punishment in the way I said I would. I wasn't kidding.
11. Other than that, have fun and good luck!

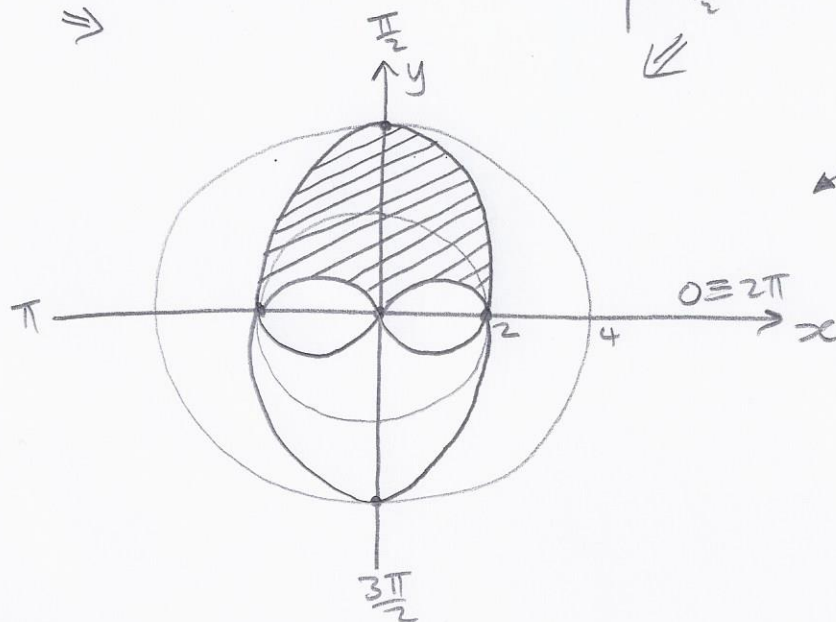
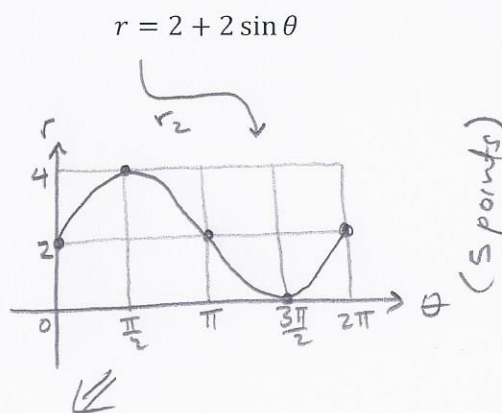
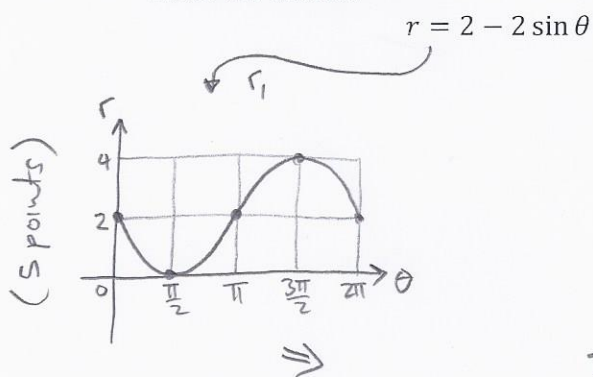
They survived to the last day of calc 2??

SOMEBODY



GIVE THAT PERSON A MEDAL!!!

1. (40 points) Sketch both polar graphs on one pair of axes, and find the area outside the first but inside the second.



(10 pts for diagram.
This includes
2 pts for shading
the right area.)

Intersections

$$2 - 2 \sin \theta = 2 + 2 \sin \theta$$

$$\Rightarrow \sin \theta = 0$$

$$\Rightarrow \theta = 0, \pi, 2\pi, \dots$$

$$\Rightarrow A = \frac{1}{2} \int R^2 - r^2$$

$$= \frac{1}{2} \int_0^\pi (2 + 2 \sin \theta)^2 - (2 - 2 \sin \theta)^2 d\theta$$

$$= \frac{1}{2} \int_0^\pi 4 + 8 \sin \theta + 4 \sin^2 \theta - 4 + 8 \sin \theta - 4 \sin^2 \theta d\theta$$

$$= 8 \int_0^\pi \sin \theta d\theta$$

$$= -8 \cos \theta \Big|_0^\pi \longrightarrow 8 \text{ pts to evaluate integral.}$$

$$= \boxed{16} \longrightarrow 2 \text{ pts for correct final answer.}$$

10 pts to set up
integral.

2. (15 points) Set up an integral for the arc length of the polar graph $r = 2 \cos 4\theta$ for $0 \leq \theta \leq \frac{\pi}{3}$.

$$L = \int_0^{\frac{\pi}{3}} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta \longrightarrow (5 \text{ pts formula})$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{(2 \cos 4\theta)^2 + (-8 \sin 4\theta)^2} d\theta \quad (5 \text{ pts set up})$$

$$= \int_0^{\frac{\pi}{3}} \sqrt{4 \cos^2 4\theta + 64 \sin^2 4\theta} d\theta$$

or

$$= \int_0^{\frac{\pi}{3}} \sqrt{4 + 60 \sin^2 4\theta} d\theta$$

(5 pts simplify)

3. (15 points) Find the angle θ needed to eliminate the xy -term in

$$x^2 + 2\sqrt{3}xy + 3y^2 + (4 + 2\sqrt{3})x + (4\sqrt{3} - 2)y + 12 = 0$$

$$\Rightarrow A=1, B=2\sqrt{3}, C=3 \longrightarrow 5 \text{ pts for identifying } A, B, C.$$

$$\begin{aligned} \cot 2\theta &= \frac{A-C}{B} \longrightarrow 5 \text{ pts for set up} \\ &= \frac{-2}{2\sqrt{3}} \end{aligned}$$

$$\Rightarrow \tan 2\theta = -\sqrt{3}$$

$$\Rightarrow 2\theta = -\frac{\pi}{3}$$

$$\Rightarrow \boxed{\theta = -\frac{\pi}{6}} \longrightarrow 5 \text{ pts to evaluate } \theta \text{ correctly.}$$

4. (30 points) Identify the conic section whose equation is $16x^2 - 9y^2 - 32x - 128 = 0$. Make a sketch, and label any of the following which may be present: vertices, foci, asymptotes.

$$16x^2 - 32x - 9y^2 - 128 = 0$$

$$\Rightarrow 16(x^2 - 2x) - 9y^2 = 128$$

$$\Rightarrow 16[x^2 - 2x + 1 - 1] - 9y^2 = 128$$

$$\Rightarrow 16[(x-1)^2 - 1] - 9y^2 = 128$$

$$\Rightarrow 16(x-1)^2 - 16 - 9y^2 = 128$$

$$\Rightarrow 16(x-1)^2 - 9y^2 = 144$$

$$\Rightarrow \frac{(x-1)^2}{9} - \frac{y^2}{16} = 1 \longrightarrow \text{Hyperbola!}$$

(10 pts to get to formula)

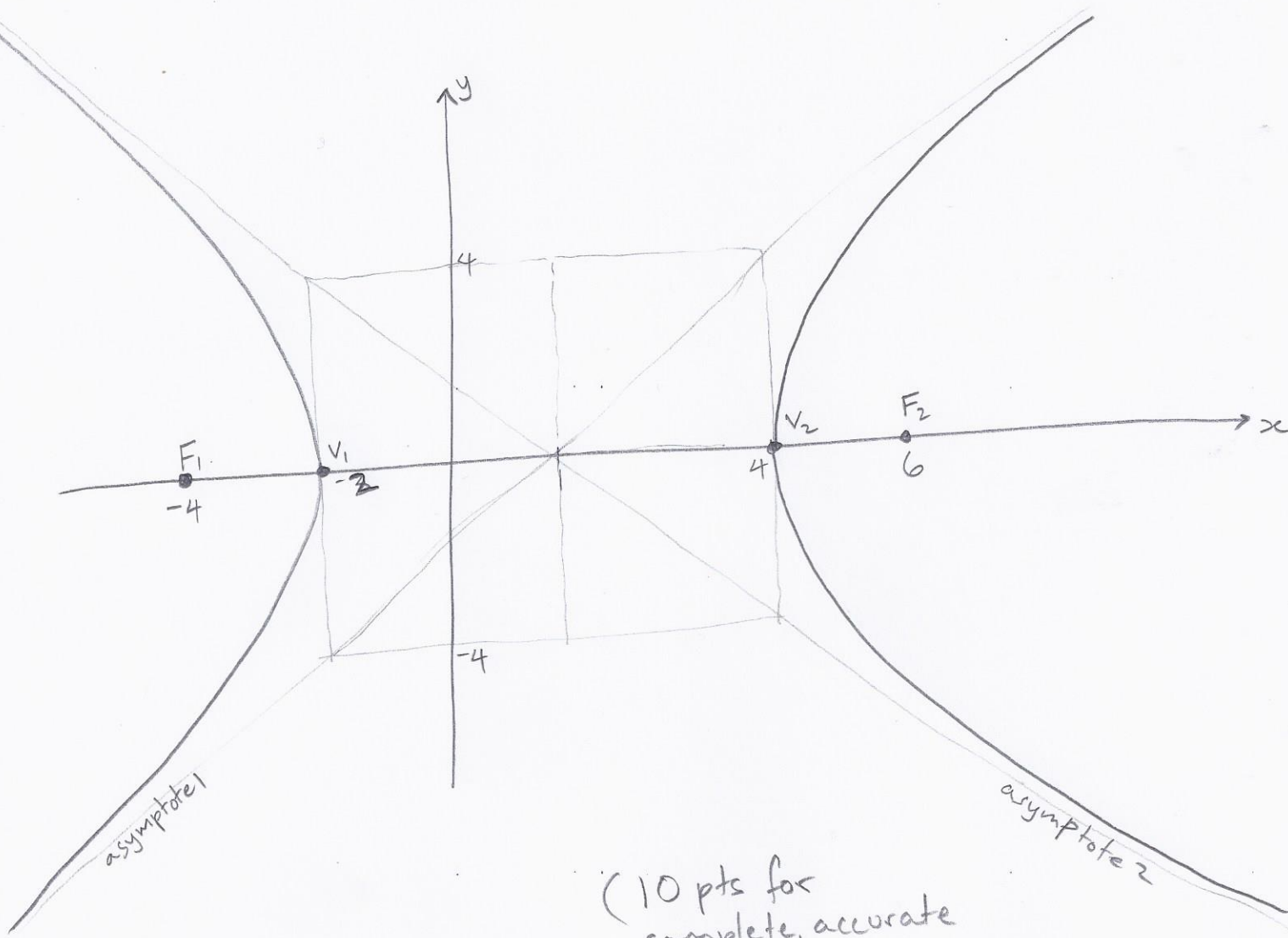
(10 pts for figuring these out)

Center $(1, 0)$

vertices $(1 \pm 3, 0)$

Foci $(1 \pm 5, 0)$

$$c^2 = a^2 + b^2 = 9 + 16 \\ \Rightarrow c = 5$$



(10 pts for complete, accurate sketch)

Bonus:

1. (5 points – all or nothing) What is

$$\int \sec^3 x \, dx =$$

$$\frac{1}{2}(\sec x \tan x + \ln|\sec x + \tan x|) + C$$

(all or nothing)

2. (5 points) Evaluate:

$$\begin{aligned} \int_0^1 \int_1^2 xy^2 \, dy \, dx \\ &= \int_0^1 \left. \frac{xy^3}{3} \right|_1^2 dx \\ &= \int_0^1 \frac{8}{3}x - \frac{1}{3}x \, dx \\ &= \frac{7}{3} \int_0^1 x \, dx \\ &= \frac{7}{3} \left. \frac{x^2}{2} \right|_0^1 \\ &= \boxed{\frac{7}{6}} \end{aligned}$$

3. (3 points) Complete the table:

θ	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\sin \theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos \theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$

(1 pt per correct column)

4. (1 point) State the integration by parts formula:

$$\int u \, dv = uv - \int v \, du$$

5. (1 point) What mnemonic is used to choose the parts?

LIATE

} all or nothing

I know when this is all said and done, you will miss me and this class terribly. Here's how to deal.

