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**Note that both sides of each page may have printed material.**

**Instructions:**

1. Read the instructions.
2. Panic!!! Kidding, don't panic! I repeat, do NOT panic!
3. Complete all problems in the actual test. Bonus problems are, of course, optional.
4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers (unless it's a case of fill in the blank or state a definition, etc.)
5. Write neatly so that I am able to follow your sequence of steps and box your answers.
6. Read through the exam and complete the problems that are easy (for you) first!
7. No scrap paper, calculators, notes or other outside aids allowed—including divine intervention, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.
8. In fact, **cell phones should be out of sight!**
9. Use the correct notation and write what you mean!  $x^2$  and  $x2$  are not the same thing, for example, and I will grade accordingly.
10. Do NOT commit any of the blasphemies or mistakes I mentioned in the syllabus. I will actually mete out punishment in the way I said I would. I wasn't kidding.
11. Other than that, have fun and good luck!

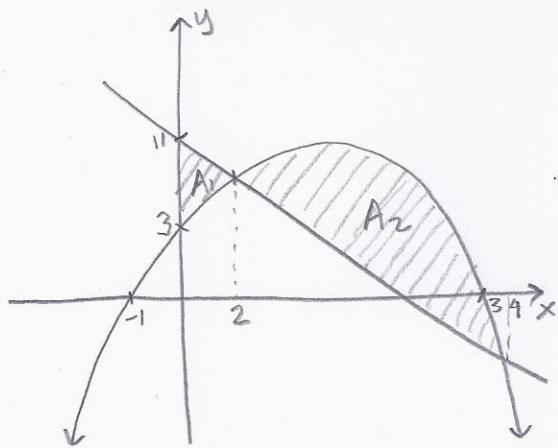
**When you're having  
fun at Thanksgiving**



**And you remember  
you have a math  
test in a couple days**



1. (25 points) Find the area bounded between the curves  $y = 3 + 2x - x^2$  and  $y = -4x + 11$  for  $0 \leq x \leq 4$ . Include a sketch of the region and shade the required area.



$$A = A_1 + A_2$$

$$\begin{aligned}
 &= \int_0^2 -4x + 11 - (3 + 2x - x^2) dx + \int_2^4 3 + 2x - x^2 - (-4x + 11) dx \\
 &= \int_0^2 8 - 6x + x^2 dx - \int_2^4 8 - 6x + x^2 dx \\
 &\Rightarrow \left[ 8x - 3x^2 + \frac{x^3}{3} \right]_0^2 - \left[ 8x - 3x^2 + \frac{x^3}{3} \right]_2^4 \\
 &= 16 - 12 + \frac{8}{3} - 0 - \left( 32 - 48 + \frac{64}{3} - \left( 16 - 12 + \frac{8}{3} \right) \right) \\
 &= 4 + \frac{8}{3} + 16 - \frac{64}{3} + 4 + \frac{8}{3} \\
 &= 24 - \frac{48}{3} \\
 &= \boxed{8}
 \end{aligned}$$

Intersection points

$$\begin{aligned}
 3 + 2x - x^2 &= -4x + 11 \\
 \Rightarrow x^2 - 6x + 8 &= 0 \\
 \Rightarrow (x-4)(x-2) &= 0 \\
 \Rightarrow x = 4, x = 2
 \end{aligned}$$

For  $y = 3 + 2x - x^2$

$$\begin{aligned}
 x\text{-int: } -(x-3)(x+1) &= 0 \\
 \Rightarrow x = 3, x = -1
 \end{aligned}$$

$$y\text{-int: } y = 3$$

For  $y = -4x + 11$

$$x\text{-int: } x = \frac{11}{4}$$

$$y\text{-int: } y = 11$$

5 pts for diagram

5 pts for intersection calculations

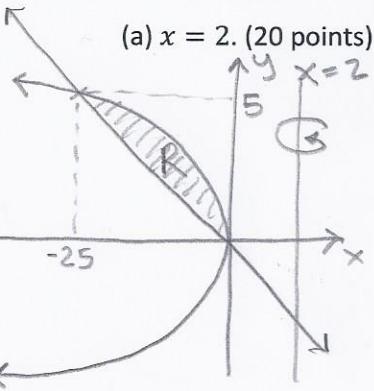
5 pts for correct integral set up

5 pts to get to fifth to last line

4 pts to plug in numbers correctly

1 pt for correct final answer

2. Let  $R$  be the region bounded by  $x = -y^2$  and  $y = -\frac{x}{5}$ . Find the volume of the solid obtained by rotating  $R$  about the line:



(a)  $x = 2$ . (20 points)

### Intersections

$$\begin{aligned} -y^2 &= -\frac{x}{5} \\ \Rightarrow y^2 &= \frac{x}{5} \\ y &= 0, y = \sqrt{\frac{x}{5}} \\ x &= 0, x = 5 \end{aligned}$$

### Using Shell Method

$$r = 2 - x$$

$$h = \sqrt{-x} + \frac{x}{5}$$

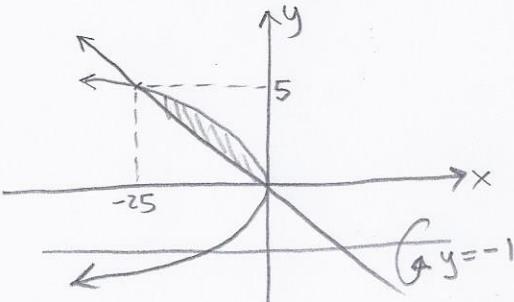
$$\begin{aligned} V &= 2\pi \int_{-25}^0 (2-x)((-x)^{1/2} + \frac{x}{5}) dx \\ &= 2\pi \int_{-25}^0 2(-x)^{1/2} + \frac{2}{5}x - (-x)^{3/2} - \frac{x^2}{5} dx \\ &= 2\pi \left[ -\frac{4}{3}(-x)^{3/2} + \frac{x^2}{5} - \frac{2}{5}(-x)^{5/2} - \frac{x^3}{15} \right] \Big|_{-25}^0 \\ &= -2\pi \left( -\frac{500}{3} + 125 - 1250 + \frac{3125}{3} \right) \\ &= \boxed{500\pi} \end{aligned}$$

Full credit  
for getting to  
this point

### Using Disk/Washer Method

$$\begin{aligned} R &= 2 + 5y, \quad r = 2 + y^2 \\ \Rightarrow V &= \pi \int_0^5 (2+5y)^2 - (2+y^2)^2 dy \\ &= \pi \int_0^5 20y + 21y^2 - y^4 dy \\ &= \pi \left[ 10y^2 + \frac{21}{3}y^3 - \frac{y^5}{5} \right] \Big|_0^5 \\ &= \pi [250 + 875 - 625] \\ &= \boxed{500\pi} \end{aligned}$$

(b)  $y = -1$ . (20 points)



Full credit for  
getting to this  
point

### Using Disk/Washer Method

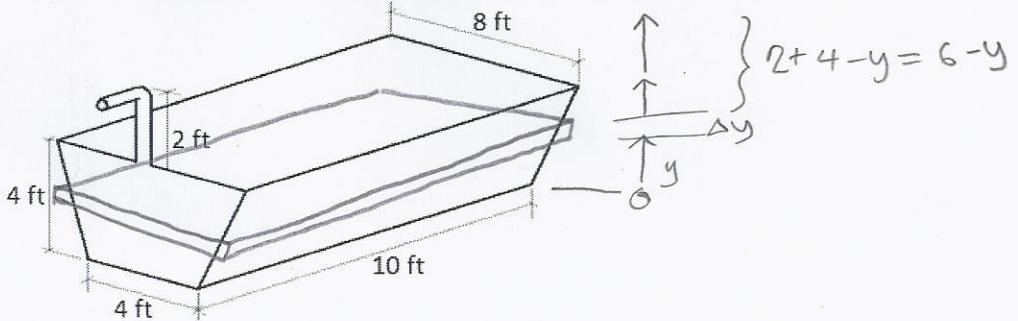
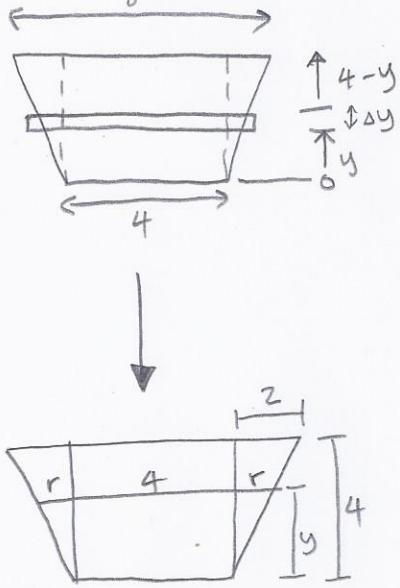
$$R = (-x)^{1/2} + 1, \quad r = -\frac{x}{5} + 1$$

$$\begin{aligned} V &= \pi \int_{-25}^0 ((-x)^{1/2} + 1)^2 - (-\frac{x}{5} + 1)^2 dx \\ &= \pi \int_{-25}^0 -\frac{3x}{5} + 2(-x)^{1/2} - \frac{x^2}{25} dx \\ &= \pi \left[ -\frac{3x^2}{10} - \frac{4}{3}(-x)^{3/2} - \frac{x^3}{75} \right] \Big|_{-25}^0 \\ &= -\pi \left[ -\frac{3(25)}{10} - \frac{4}{3}(25)^{3/2} - \frac{(25)^3}{75} \right] \\ &= \boxed{\frac{875\pi}{6}} \end{aligned}$$

### Using Shell Method

$$\begin{aligned} r &= y+1, \quad h = -y^2 + 5y \\ V &= 2\pi \int_0^5 (y+1)(-y^2 + 5y) dy \\ &= 2\pi \int_0^5 -y^3 + 4y^2 + 5y dy \\ &= 2\pi \left[ -\frac{y^4}{4} + \frac{4}{3}y^3 + \frac{5}{2}y^2 \right] \Big|_0^5 \\ &= 2\pi \left[ -\frac{5}{4} + \frac{4}{3}(5)^3 + \frac{5}{2}(5)^2 \right] \\ &= \boxed{\frac{875\pi}{6}} \end{aligned}$$

3. (25 points) A tank has the shape of a trapezoidal prism as shown below. It is filled to a level of 2 ft with a liquid having density 20 lb per cubic foot. The tank is 10 ft long, 4 ft high, has base width 4 ft and top width 8 ft and a 2 ft spout. Find the work needed to pump all the liquid out the spout.



$$\begin{aligned}
 W_{\text{slice}} &= \text{Force} \times \text{Distance} \\
 &= \text{Volume} \times \text{density} \times \text{force} \times \text{distance} \\
 &= (\text{length} \times \text{width} \times \text{height}) \times \text{density} \times \text{distance} \\
 &= 10(4+y)\Delta y \times 20 \times (6-y)
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow W_{\text{total}} &= \int_0^2 200(4+y)(6-y) dy \\
 &= 200 \int_0^2 24 + 2y - y^2 dy \\
 &= 200 [24y + y^2 - \frac{y^3}{3}] \Big|_0^2 \\
 &= 200 [24(2) + 2^2 - \frac{2^3}{3}] \\
 &= \boxed{\frac{29600}{3} \text{ ft-lb}}
 \end{aligned}$$

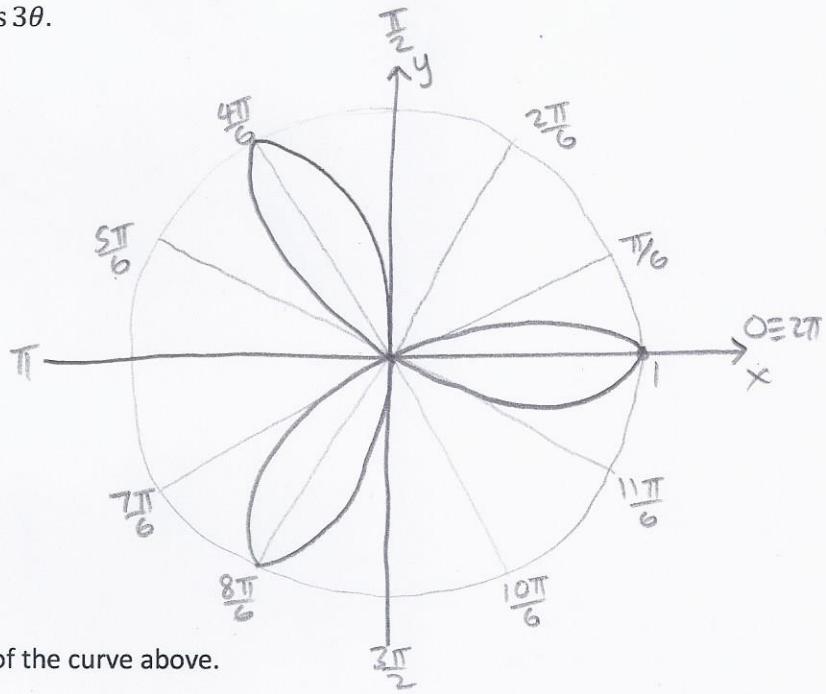
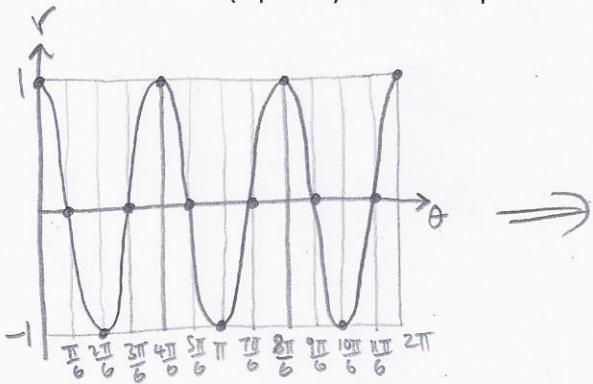
Full credit for getting to here

- 5 pts for diagram or figuring out the width.
- 10 pts for setting up correct integral
- 8 pts for evaluating integral
- 2 pts for getting to point shown.

4. (10 points) Find the arclength of the curve  $y = \ln \sec x$  between  $x = 0$  and  $x = \frac{\pi}{4}$ .

$$\begin{aligned}
 y' &= \tan x \\
 \Rightarrow L &= \int_0^{\frac{\pi}{4}} \sqrt{1 + \tan^2 x} dx \quad \text{——— 5 pts to set up} \\
 &= \int_0^{\frac{\pi}{4}} \sec x dx \\
 &= \ln |\sec x + \tan x| \Big|_0^{\frac{\pi}{4}} \quad \text{——— 3 pts for correct antiderivative} \\
 &= \ln |\sec \frac{\pi}{4} + \tan \frac{\pi}{4}| - \ln |1| \quad \text{——— 1 pt to plug in correctly} \\
 &= \boxed{\ln(\sqrt{2} + 1)} \quad \text{——— 1 pt for final ans.}
 \end{aligned}$$

Bonus 1: (5 points) Sketch the polar curve  $r = \cos 3\theta$ .



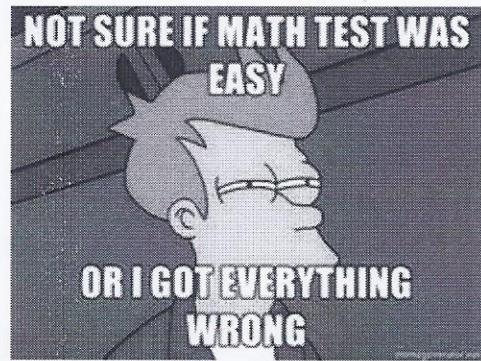
Bonus 2: (5 points) Find the area inside one loop of the curve above.

$$\begin{aligned}
 A &= \frac{1}{2} \int r^2 d\theta \\
 &= 2 \cdot \frac{1}{2} \int_0^{\pi/6} \cos^2 3\theta d\theta \\
 &= \frac{1}{2} \int_0^{\pi/6} 1 + \cos 6\theta d\theta \\
 &= \frac{1}{2} \theta \Big|_0^{\pi/6} \\
 &= \boxed{\frac{\pi}{12}}
 \end{aligned}$$

Bonus 3: (5 points) Fully identify the conic section:  $9x^2 + 16y^2 - 18x + 64y = 71$

$$\begin{aligned}
 9(x^2 - 2x) + 16(y^2 + 4y) &= 71 \\
 \Rightarrow 9(x^2 - 2x + 1 - 1) + 16(y^2 + 4y + 4 - 4) &= 71 \\
 \Rightarrow 9[(x-1)^2 - 1] + 16[(y+2)^2 - 4] &= 71 \\
 \Rightarrow 9(x-1)^2 - 9 + 16(y+2)^2 - 64 &= 71 \\
 \Rightarrow 9(x-1)^2 + 16(y+2)^2 &= 144 \\
 \Rightarrow \frac{(x-1)^2}{16} + \frac{(y+2)^2}{9} &= 1
 \end{aligned}$$

$\Rightarrow$  Ellipse with center  $(1, -2)$ , horizontal major axis, length 8;  
vertical minor axis, length 6



NOT SURE IF MATH TEST WAS  
EASY

OR I GOT EVERYTHING  
WRONG