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Note that both sides of each page may have printed material.

Instructions:

1. Read the instructions.
2. Panic!!! Kidding, don't panic! I repeat, do NOT panic!
3. Complete all problems in the actual test. Bonus problems are, of course, optional.
4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers (unless it's a case of fill in the blank or state a definition, etc.)
5. Write neatly so that I am able to follow your sequence of steps and box your answers.
6. Read through the exam and complete the problems that are easy (for you) first!
7. No scrap paper, calculators, notes or other outside aids allowed—including divine intervention, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.
8. In fact, **cell phones should be out of sight!**
9. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.
10. Do NOT commit any of the blasphemies or mistakes I mentioned in the syllabus. I will actually mete out punishment in the way I said I would. I wasn't kidding.
11. Other than that, have fun and good luck!

Remember: Avoid confusion

WHAT IF I TOLD YOU



THIS IS NOT A TEST?

1. Compute the following integrals (10 points each):

$$(a) \int 3x^3 \cos x \, dx$$

\oplus	$9x^2$	$\sin x$	$\rightarrow 3x^3 \sin x$
\ominus	$18x$	$-\cos x$	$\rightarrow 9x^2 \cos x$
\oplus	18	$-\sin x$	$\rightarrow -18x \sin x$
\ominus	0	$\cos x$	$\rightarrow -18 \cos x$

$$\Rightarrow \int 3x^3 \cos x \, dx = \boxed{3x^3 \sin x + 9x^2 \cos x - 18x \sin x - 18 \cos x + C}$$

$$(b) \int \sqrt{\tan x} \sec^6 x \, dx$$

$$u = \tan x$$

$$du = \sec^2 x \, dx$$

$$\begin{aligned} & \int u^{1/2} (1+u^2)^2 \, du \\ &= \int u^{1/2} (1+2u^2+u^4) \, du \\ &= \int u^{1/2} + 2u^{5/2} + u^{9/2} \, du \\ &= \frac{2}{3}u^{3/2} + \frac{4}{7}u^{7/2} + \frac{2}{11}u^{11/2} + C \end{aligned}$$

$$= \boxed{\frac{2}{3}(\tan x)^{3/2} + \frac{4}{7}(\tan x)^{7/2} + \frac{2}{11}(\tan x)^{11/2} + C}$$

$$(c) \int_1^e x^{5/2} \ln x \, dx$$

$$u = \ln x \quad dv = x^{5/2} dx \\ du = \frac{1}{x} dx \quad v = \frac{2}{7} x^{7/2}$$

$$\int x^{5/2} \ln x \, dx = \frac{2}{7} x^{7/2} \ln x - \frac{2}{7} \int x^{5/2} dx \\ = \frac{2}{7} x^{7/2} \ln x - \frac{4}{49} x^{7/2} + C$$

$$\Rightarrow \int_1^e x^{5/2} \ln x \, dx = \frac{2}{7} e^{7/2} - \frac{4}{49} e^{7/2} + \frac{4}{49}$$

$$= \boxed{\frac{10}{49} e^{7/2} + \frac{4}{49}}$$

$$(d) \int \frac{x^2 - 6x + 1}{x^2 - 6x - 7} \, dx$$

$$\int \frac{x^2 - 6x - 7 + 8}{x^2 - 6x - 7} \, dx$$

$$= \int \left(1 + \frac{8}{(x-7)(x+1)} \right) dx$$

$$= \int \left(1 + \frac{1}{x-7} - \frac{1}{x+1} \right) dx$$

$$= \boxed{x + \ln|x-7| - \ln|x+1| + C}$$

or

$$= \boxed{x + \ln \left| \frac{x-7}{x+1} \right| + C}$$

$$(e) \int \frac{\pi x^2}{\sqrt{4-x^2}} dx$$

$$x = 2 \sin \theta \\ dx = 2 \cos \theta d\theta$$

$$\pi \int \frac{4 \sin^2 \theta \cdot 2 \cos \theta}{2 \cos \theta} d\theta$$

$$= 2\pi \int (1 - \cos 2\theta) d\theta$$

$$= 2\pi \left(\theta - \frac{1}{2} \sin 2\theta \right) + C$$

$$= 2\pi \left(\theta - \sin \theta \cos \theta \right) + C$$

$$= \boxed{2\pi \left(\sin^{-1} \frac{x}{2} - \frac{1}{4} x \sqrt{4-x^2} \right) + C}$$

$$\frac{x}{2} = \sin \theta \\ \Rightarrow \begin{array}{c} 2 \\ \theta \\ \sqrt{4-x^2} \end{array} \quad x$$

$$\Rightarrow \cos \theta = \frac{1}{2} \sqrt{4-x^2}$$

$$\Rightarrow \theta = \sin^{-1} \left(\frac{x}{2} \right)$$

2. Write down the partial fractions decomposition of the following. Do not attempt to solve for the arbitrary constants (2 points each):

$$(a) \frac{5x^3 - 7}{x^2(x+7)(x^2+7)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+7} + \frac{Dx+E}{x^2+7} + \frac{Fx+G}{(x^2+7)^2}$$

$$(b) \frac{5x}{x^3(x^2+4)^2(x^2-9)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+4} + \frac{Ex+F}{(x^2+4)^2} + \frac{Gx+H}{x-3} + \frac{Ix+J}{x+3}$$

$$(c) \frac{x^2+4}{x^2-4} = 1 + \frac{8}{x^2-4} = 1 + \frac{A}{x-2} + \frac{B}{x+2}$$

$$(d) \frac{\pi - ex^3}{x^6 - x^3} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-1} + \frac{Ex+F}{x^2+x+1}$$

$$(e) \frac{x(x^2+4x+4)}{x^2(x^2+4x+4)(x+3)} = \frac{A}{x} + \frac{B}{x+3}$$

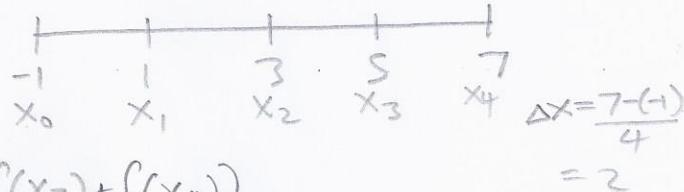
3. Set up (you should leave your answer as a sum of fractions) the approximations for

$$\int_{-1}^7 \frac{1}{2x+7} dx$$

using $n = 4$ and:

(a) The Trapezoidal rule (5 points):

$$\begin{aligned} T_4 &= \frac{\Delta x}{2} (f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)) \\ &= 1 \cdot \left(\frac{1}{-2+7} + \frac{2}{-1+7} + \frac{2}{1+7} + \frac{2}{6+7} + \frac{1}{14+7} \right) \\ &= \boxed{\left(\frac{1}{5} + \frac{2}{9} + \frac{2}{13} + \frac{2}{17} + \frac{1}{21} \right)} \end{aligned}$$



(b) Simpson's rule (5 points):

$$\begin{aligned} S_4 &= \frac{\Delta x}{3} (f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)) \\ &= \boxed{\frac{2}{3} \left(\frac{1}{5} + \frac{4}{9} + \frac{2}{13} + \frac{4}{17} + \frac{1}{21} \right)} \end{aligned}$$

(c) What is the error in the Trapezoidal rule approximation here? (5 points)

$$\begin{aligned} E_T &= \frac{K(b-a)^3}{12n^2} \\ &= \boxed{\frac{\frac{8}{125}(8)^3}{12 \cdot 4^2}} \end{aligned}$$

$$\begin{aligned} f &= (2x+7)^{-1} \\ \Rightarrow f' &= -2(2x+7)^{-2} \\ \Rightarrow f'' &= 8(2x+7)^{-3} \\ \Rightarrow |f''| &= \left| \frac{8}{(2x+7)^3} \right| \\ &\leq \frac{8}{5^3} = \frac{8}{125} \text{ on } [-1, 7] \end{aligned}$$

(d) What must n be to ensure the Trapezoidal approximation is accurate to at least 0.01? (5 points)

$$\text{Want } |E_T| \leq 0.01$$

$$\Rightarrow \frac{\frac{8^4}{125}}{12 \cdot n^2} \leq \frac{1}{100}$$

$$\Rightarrow \frac{12n^2}{\frac{8^4}{125}} \geq 100$$

$$\Rightarrow n \geq \sqrt{\frac{100 \cdot 8^4}{12 \cdot 125}}$$

n must be the smallest integer larger than the number

4. For each of the integrals below: if the integral is convergent, find its value; if it is divergent, show why. (10 points each)

$$(a) \int_0^\infty 2xe^{-2x} dx$$

$$\begin{aligned} u &= 2x & dv &= e^{-2x} dx \\ du &= 2dx & v &= -\frac{1}{2}e^{-2x} \end{aligned}$$

$$\Rightarrow \int 2xe^{-2x} dx = -xe^{-2x} + \int e^{-2x} dx \\ = -xe^{-2x} - \frac{1}{2}e^{-2x} + C$$

$$\Rightarrow \int_0^\infty 2xe^{-2x} dx = \lim_{N \rightarrow \infty} \left(-xe^{-2x} - \frac{1}{2}e^{-2x} \right) \Big|_0^N \\ = \lim_{N \rightarrow \infty} \left(-Ne^{-2N} - \frac{1}{2}e^{-2N} + \frac{1}{2} \right) \\ = \frac{1}{2}$$

converges to $\frac{1}{2}$

$$(b) \int_0^\infty \frac{3}{5x-7} dx$$

$$\int_0^\infty \frac{3}{5x-7} dx = \int_0^{\frac{7}{5}} \frac{3}{5x-7} dx + \int_{\frac{7}{5}}^2 \frac{3}{5x-7} dx + \int_2^\infty \frac{3}{5x-7} dx \\ = \lim_{N \rightarrow \frac{7}{5}^-} \int_0^N \frac{3}{5x-7} dx + \lim_{M \rightarrow \frac{7}{5}^+} \int_M^2 \frac{3}{5x-7} dx + \lim_{P \rightarrow \infty} \int_2^P \frac{3}{5x-7} dx \\ = \lim_{N \rightarrow \frac{7}{5}^-} \frac{3}{5} \left| \ln|5x-7| \right|_0^N + \lim_{M \rightarrow \frac{7}{5}^+} \frac{3}{5} \left| \ln|5x-7| \right|_M^2 + \lim_{P \rightarrow \infty} \frac{3}{5} \left| \ln|5x-7| \right|_2^P$$

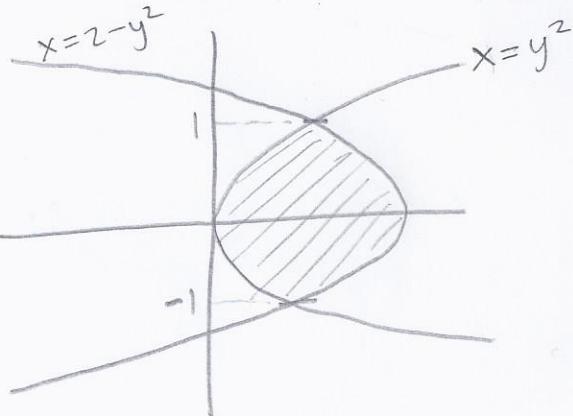
$$\downarrow \\ \lim_{N \rightarrow \frac{7}{5}^-} \left(\frac{3}{5} \ln|5N-7| - \frac{3}{5} \ln|-7| \right)$$

$\rightarrow -\infty$

\Rightarrow the integral diverges

Bonus Problems: Set up integrals to find the area of the region bounded by the indicated curves.

(a) $x = y^2$ and $x = 2 - y^2$, include a sketch and shade the described region (5 points)

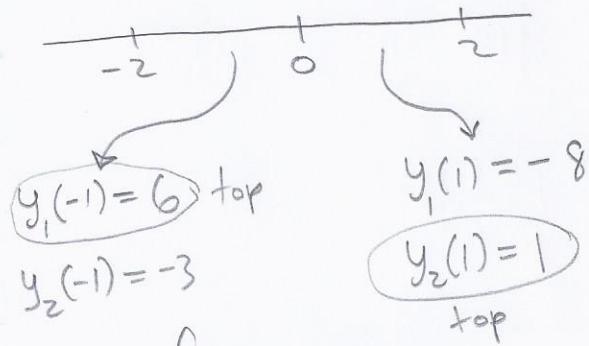


$$\begin{aligned} & \text{Intersections} \\ & y^2 = 2 - y^2 \\ & \Rightarrow y = \pm 1 \end{aligned}$$

$$\begin{aligned} A &= \int_{\text{right}} - \int_{\text{left}} \\ &= \int_{-1}^1 (2 - y^2 - y^2) dy \\ &= \boxed{\int_{-1}^1 (2 - 2y^2) dy} \end{aligned}$$

(b) $y = 3x^3 - x^2 - 10x$ and $y = -x^2 + 2x$. Show how you got your points of intersection and how you chose which function to put first. (10 points)

$$\begin{aligned} & \text{Intersections} \\ & 3x^3 - x^2 - 10x = -x^2 + 2x \\ & \Rightarrow 3x^3 - 12x = 0 \\ & \Rightarrow 3x(x^2 - 4) = 0 \\ & \Rightarrow x = 0, x = \pm 2 \end{aligned}$$



$$\begin{aligned} A &= \int_{\text{top}} - \int_{\text{bottom}} \\ &= \int_{-2}^0 3x^3 - x^2 - 10x - (-x^2 + 2x) dx + \int_0^2 -x^2 + 2x - (3x^3 - x^2 - 10x) dx \\ &= \boxed{\int_{-2}^0 (3x^3 - 12x) dx - \int_0^2 (3x^3 - 12x) dx} \end{aligned}$$

Remember: Math is fun, math is beautiful, this test was not hard



And there is no spoon.

I took the red pill...