Math 201 Test 3A
November 25, 2014

Name: JHEVON SMITH

Note that both sides of each page may have printed material.

Instructions:

1. Read the instructions.
2. Don't panic! I repeat, do NOT panic!
3. Complete all problems. In this exam, each non-bonus problem is worth 20 points. The weight of the bonus problems are indicated.
4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers (unless it's a case of fill in the blank or state a definition, etc.)
5. Write neatly so that I am able to follow your sequence of steps and box your answers.
6. Read through the exam and complete the problems that are easy (for you) first!
7. No scrap paper, calculators, notes or other outside aids allowed—including divine intervention, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting. In fact, **cell phones should be out of sight!**
8. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.
9. Other than that, have fun and good luck!

May the odds be ever in your favor. Muhuhahahahahaha!

$f(x) = x^3 - x^2 - 2x$

1. (a) Find the c guaranteed by the Mean Value Theorem for the function $f(x) = x(x^2 - x - 2)$ on the interval $[-1,1]$. What conditions allow you to apply the Mean Value Theorem here? (8 points)

By the MVT, there exists $c \in (-1,1)$ such that $f'(c) = \frac{f(1) - f(-1)}{1-(-1)}$

$\Rightarrow 3c^2 - 2c - 2 = \frac{-2-0}{2}$

$\Rightarrow 3c^2 - 2c - 1 = 0$

$\Rightarrow (3c+1)(c-1) = 0$

$\Rightarrow \boxed{c = -1/3}$ or $c = 1 \rightarrow$ not in $(-1,1)$

We can apply the MVT since f is continuous on $[-1,1]$ and differentiable on $(-1,1)$ (since f is a polynomial.)

(b) Find the absolute extrema of the function $f(x) = 3x^4 - 4x^3$ on $[-1,2]$. (8 points)

$f' = 12x^3 - 12x$

For crit. pts: $12x(x^2-1) = 0$

$\Rightarrow x = 0, 1, -1$

We have:

$f(-1) = 7$

$f(0) = 0$

$\boxed{f(1) = -1} \rightarrow$ abs min

$\boxed{f(2) = 16} \rightarrow$ abs max

(c) Compute $\sum_{i=1}^{n}(i+1)(i-2)$. (4 points)

$= \sum_1^n i^2 - i - 2$

$= \sum_1^n i^2 - \sum_1^n i - \sum_1^n 2$

$= \boxed{\frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} - 2n}$

2. Compute the following antiderivatives. (5 points each)

(a) $\int \frac{x^3+2x^2-9}{x^2}\,dx = \int x + 2 - 9x^{-2}\,dx$

$= \boxed{\frac{x^2}{2} + 2x + 9x^{-1} + C}$

(b) $\int (\sqrt{x}+1)^2\,dx = \int x + 2x^{1/2} + 1\,dx$

$= \boxed{\frac{x^2}{2} + \frac{4}{3}x^{3/2} + x + C}$

(c) $\int 3\cos^2 x - 3\sin^2 x\,dx = 3\int \cos 2x\,dx$

$3(\cos^2 x - \sin^2 x)$

$= \boxed{\frac{3}{2}\sin 2x + C}$

(d) $\int \sin\left(\frac{\pi}{4}x\right)\,dx = \boxed{-\frac{4}{\pi}\cos\left(\frac{\pi}{4}x\right) + C}$

3. For the function $f(x) = \frac{x^3}{x^2-3}$, find (provided they exist) the domain, intercepts, asymptotes, local extrema, inflections point(s), intervals of increasing and decreasing, and intervals of concavity. Use these to sketch the graph of $f(x)$ and indicated them on your graph. For this problem, you may assume $f'(x) = \frac{x^2(x^2-9)}{(x^2-3)^2}$ and $f''(x) = \frac{6x(x^2+9)}{(x^2-3)^3}$ (you need not verify these). (20 points)

$\boxed{\text{Domain: } (-\infty,-\sqrt{3})\cup(-\sqrt{3},\sqrt{3})\cup(\sqrt{3},\infty)}$

Intercepts:

x-int $x=0$

y-int $y=0$

Asymptotes: VA

$x^2-3=0$

$\Rightarrow \boxed{x=\pm\sqrt{3} \text{ are VAs}}$

HA

$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = -\infty$

$\boxed{\text{No HA}}$

Inc/Dec/max/min:

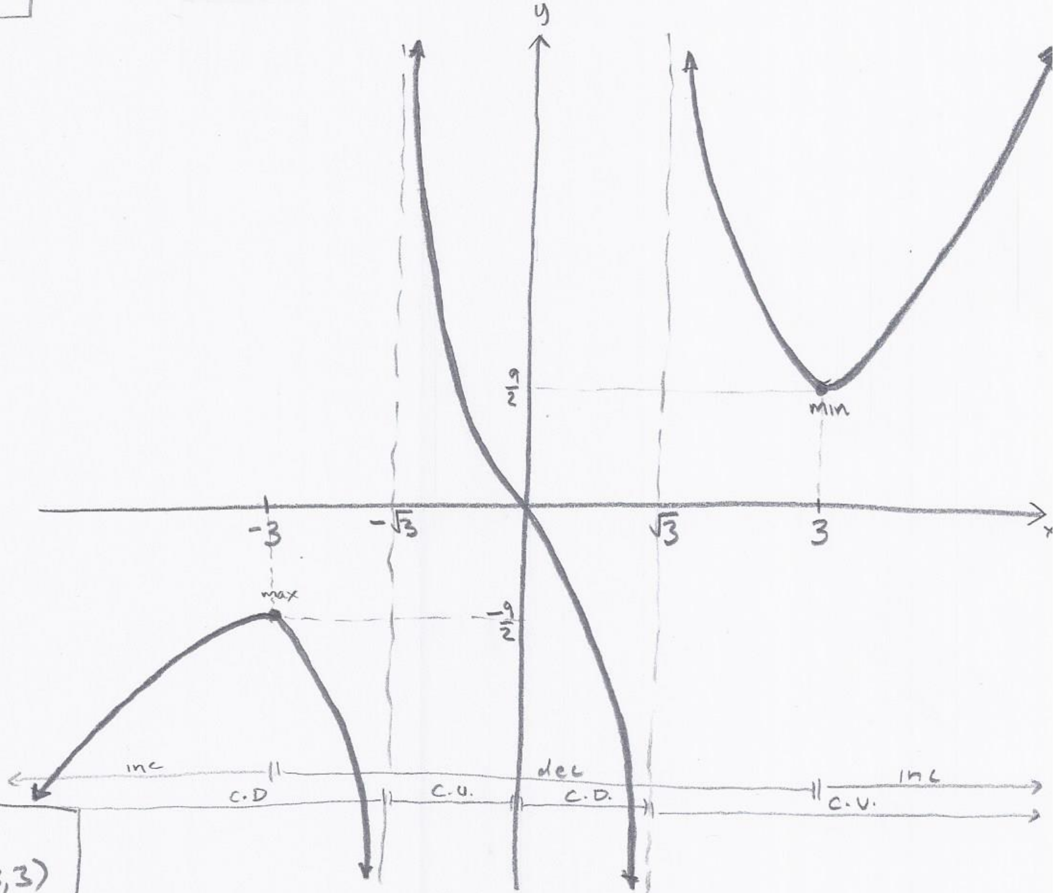
$x=0, \pm 3, \pm\sqrt{3}$

(+) (−) (−) (−) (−) (+) f'

-3 $-\sqrt{3}$ 0 $\sqrt{3}$ 3

max min

Increasing: $(-\infty,-3)\cup(3,\infty)$
Decreasing: $(-3,-\sqrt{3})\cup(-\sqrt{3},0)\cup(0,\sqrt{3})\cup(\sqrt{3},3)$
max: $(-3,-9/2)$
min: $(3,9/2)$



Concavity/Inflections:

$x=0, \pm\sqrt{3}$

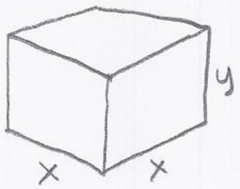
(−) (+) (−) (+) f''

$-\sqrt{3}$ 0 $\sqrt{3}$

inf

Concave up: $(-\sqrt{3},0)\cup(\sqrt{3},\infty)$
Concave down: $(-\infty,-\sqrt{3})\cup(0,\sqrt{3})$
Inflection: $(0,0)$

4. A storage shed is to be built in the shape of a box with a square base. It is to have a volume of 300 cubic feet. The concrete for the base costs $8 per square foot, the material for the roof costs $4 per square foot, and the material for the sides cost $5 per square foot. Find the dimensions of the most economical shed. (20 points)



Constraint

$x^2y = 300$

$\Rightarrow y = \frac{300}{x^2}$

Objective

$C = \underbrace{8x^2}_{\text{base}} + \underbrace{4x^2}_{\text{roof}} + \underbrace{5(4xy)}_{\text{sides}}$

$C = 12x^2 + 20xy$

$\Rightarrow C = 12x^2 + 20x\left(\frac{300}{x^2}\right)$

$\Rightarrow C = 12x^2 + 6000x^{-1}$

$\Rightarrow C' = 24x - 6000x^{-2}$

For min C, set $C' = 0$.

$\Rightarrow 24x - \frac{6000}{x^2} = 0$

$\Rightarrow 24x^3 - 6000 = 0$

$\Rightarrow x = \sqrt[3]{\frac{6000}{24}}$

$= \sqrt[3]{250}$

$= 5\sqrt[3]{2}$

$\Rightarrow y = \frac{300}{25\sqrt[3]{4}} = 6\sqrt[3]{2}$

$\Rightarrow$ The dimensions are
length $= 5\sqrt[3]{2}$ ft
width $= 5\sqrt[3]{2}$ ft
height $= 6\sqrt[3]{2}$ ft

5. A moving particle has an acceleration given by $a(t) = 3t^3$. If its initial velocity is $v(0) = 1$ and its initial position is $s(0) = 0$, find the a position function, $s(t)$, and the velocity function, $v(t)$, for the object. Does the object ever change direction? How do you know? (20 points)

$a(t) = 3t^3$

$\Rightarrow v(t) = \frac{3}{4}t^4 + C$

Since $v(0) = 1$

$\Rightarrow 1 = C$

$\Rightarrow \boxed{v(t) = \frac{3}{4}t^4 + 1}$

$\Rightarrow s(t) = \frac{3}{20}t^5 + t + C$

Since $s(0) = 0$

$\Rightarrow 0 = C$

$\Rightarrow \boxed{s(t) = \frac{3}{20}t^5 + t}$

The object never changes direction.
We know because the velocity is always positive

Bonus Problems: (You must complete all problems in the actual test to be eligible).

1. Compute $\lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{i}{n}\right)^2 \left(\frac{1}{n}\right)\right]$. (5 points)

$$= \lim_{n\to\infty} \frac{1}{n^3} \sum_{1}^{n} i^2$$

$$= \lim_{n\to\infty} \frac{1}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$= \boxed{\frac{1}{3}}$$

2. (a) Use right hand endpoints and four subintervals to approximate the area under the curve $f(x) = x^2 + 1$ between $x = 1$ and $x = 5$. (5 points)

$$\Delta x = \frac{5-1}{4} = 1$$

$$\Rightarrow A \approx R_4 = \Delta x\left(f(2)+f(3)+f(4)+f(5)\right)$$

$$= 1\cdot\left(2^2+1+3^2+1+4^2+1+5^2+1\right)$$

$$= 4+4+9+16+25$$

$$\boxed{= 58}$$

(b) Compute the exact area that was approximated in part (a) by a method of your choice. (5 points)

Best: $A = \int_1^5 x^2+1\,dx$

$$= \frac{x^3}{3}+x\Big|_1^5$$

$$= \frac{5^3}{3}+5-\left(\frac{1}{3}+1\right)$$

$$= \boxed{\frac{136}{3}}$$

(c) Write down the limit of a Riemann sum to compute the exact area found in part (b). (5 points) $\Delta x = \frac{5-1}{n} = \frac{4}{n}$, $x_i = 1+\frac{4i}{n}$

$$\Rightarrow A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \boxed{\lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(1+\frac{4i}{n}\right)^2+1\right]\left(\frac{4}{n}\right) = A}$$