Math 201 Quiz 7B

October 24, 2014

Name: ANSWER

Instructions: No calculators. Use your own scrap. Write your fully simplified answers in the space provided. Assume all given functions ae differentiable.

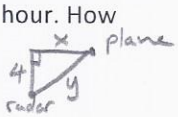
1. Suppose $y = f(x)$, find the differential $dy =$ $f'(x)\,dx$

2. For a function $f(x)$ write down the formula for its linearization at a. $L(x) =$ $f(a) + f'(a)(x-a)$

3. Suppose $x^2y - 6xy^2 + (9+\pi)y = \pi$, find:
   (a) $\frac{dy}{dx} =$ $\dfrac{-2xy - 6y^2}{x^2 - 12xy + 9 + \pi}$ (b) $\frac{dx}{dy} =$ $-\dfrac{x^2 - 12xy + 9 + \pi}{2xy - 6y^2}$
   (c) The equation of the tangent line when $y = 1$. $y = 1$

4. Use linear approximation or differentials to approximate $(27.1)^{\frac{2}{3}}$ by completing the following:
   (a) Define a function to use: $f(x) =$ $x^{2/3}$
   (b) $x =$ 27.1, $a =$ 27
   (c) The formula used to make the approximation $f(x) \approx f(a) + f'(a)(x-a)$ (may plug in $x = 27.1$ and $a = 27$)
   (d) The approximate value is $9 + 1/45 = 406/45$

5. An airplane is flying at an altitude of 4 miles and passes directly over a radar antenna. When the distance between the plane and the antenna is 8 miles, the radar detects that this distance is changing at a rate of 240 miles per hour. How fast is the plane flying?
   The equation I used (before differentiating) is $y^2 = x^2 + 4^2$ (or something similar)



   After differentiating, I have $2y\frac{dy}{dt} = 2x\frac{dx}{dt}$
   The plane is traveling at a speed of $480/\sqrt{3}$ mph

6. A 6-foot tall man is walking at a rate of 4 feet per second away from a light that is 12 feet above the ground. When he is 8 feet from the base of the light,
   (a) At what rate is the tip of his shadow moving? $5 + 5 = 10$ ft/sec
   (b) At what rate is the length of his shadow changing? 5 ft/sec

7. A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a rate of $2/\pi$ feet per second. At what rate is the total area A of disturbed water changing when $r = 3$?
   State your answer as an equation involving a derivative. $\frac{dA}{dt} = 12$

Bonus (Complete the other problems to be eligible):

(a) For a function $f(x)$, define "critical number of f" An x-value such that $f'(x) = 0$ or $f'(x)$ is undefined.

(b) Suppose a function is defined on a closed interval $[a, b]$, define the "absolute minimum of f on $[a, b]$"
The smallest value of f on $[a, b]$