Math 201 Quiz 7A

October 24, 2014

Name: ANSWERS

Instructions: No calculators. Use your own scrap. Write your fully simplified answers in the space provided. Assume all given functions ae differentiable.

1. For a function $f(x)$ write down the formula for its linearization at a. $L(x) =$ $f(a) + f'(a)(x-a)$

2. Suppose $y = f(x)$, find the differential $dy =$ $f'(x)\,dx$

3. Suppose $x^2y - 4xy^2 + (4+\pi^2)y = \pi^2$, find:

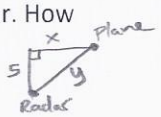
   (a) $\frac{dy}{dx} =$ $-\frac{2xy - 4y^2}{x^2 - 8xy + 4 + \pi^2}$ (b) $\frac{dx}{dy} =$ $-\frac{x^2 - 8xy + 4 + \pi^2}{2xy - 4y^2}$

   (c) The equation of the tangent line when $y = 1$. $y = 1$

4. A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a rate of π feet per second. At what rate is the total area A of disturbed water changing when $r = 2$?

   State your answer as an equation involving a derivative. $\frac{dA}{dt} = 4\pi^2$

5. An airplane is flying at an altitude of 5 miles and passes directly over a radar antenna. When the distance between the plane and the antenna is 10 miles, the radar detects that this distance is changing at a rate of 240 miles per hour. How fast is the plane flying?

   The equation I used (before differentiating) is $y^2 = x^2 + 5^2$ (or something similar)

   After differentiating, I have $2y\frac{dy}{dt} = 2x\frac{dx}{dt}$

   The plane is traveling at a speed of $480/\sqrt{3}$ mph

6. A 6-foot tall man is walking at a rate of 5 feet per second away from a light that is 15 feet above the ground. When he is 10 feet from the base of the light,

   (a) At what rate is the tip of his shadow moving? $5 + \frac{10}{3} = 25/3$ ft/sec

   (b) At what rate is the length of his shadow changing? $10/3$ ft/sec

7. Use linear approximation or differentials to approximate $(8.1)^{\frac{2}{3}}$ by completing the following:

   (a) Define a function to use: $f(x) =$ $x^{2/3}$

   (b) $x =$ 8.1, $a =$ 8

   (c) The formula used to make the approximation $f(x) \approx f(a) + f'(a)(x-a)$ (may plug in $x=8.1$, $a=8$)

   (d) The approximate value is $4 + 1/30 = 121/30$

Bonus (Complete the other problems to be eligible):

(a) For a function $f(x)$, define "critical number of f" An x-value such that $f'(x) = 0$ or $f'(x)$ is undefined.

(b) Suppose a function is defined on a closed interval $[a, b]$, define the "absolute minimum of f on $[a, b]$"

The smallest value of f on $[a,b]$.