Math 201 Quiz 6B

October 14, 2014

Name: ANSWERS

Instructions: No calculators. Use your own scrap. Write your fully simplified answers in the space provided. Assume all given functions ae differentiable.

1. Complete the following formulas (you may write f to mean $f(x)$ and g to mean $g(x)$):

   (a) $\frac{d}{dx}f(g(x)) =$ $f'(g(x))\cdot g'(x)$ (b) $\frac{d}{dx}(f(x)\cdot g(x)) =$ $f'g + fg'$ (c) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) =$ $\frac{f'g - fg'}{g^2}$

   (d) $\frac{d}{dx}x^n =$ nx^{n-1} (e) $\frac{d}{dx}\sec x =$ $\sec x \tan x$ (f) $\frac{d}{dx}\cot x =$ $-\csc^2 x$

2. Use limits to define the derivative of a function $f(x)$: $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ or $f'(a) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$

3. Use problem 2 to find the derivative of $f(x) = \frac{x}{3-4x}$ by completing the following:

   (i) Set up the limit: $f'(x) = \lim_{h\to 0}\frac{\frac{x+h}{3-4(x+h)} - \frac{x}{3-4x}}{h}$ or $\lim_{x\to a}\frac{\frac{x}{3-4x} - \frac{a}{3-4a}}{x-a}$ → $f'(a)$

   (ii) Fully simplify the limit (what you would have just before taking the limit): $\lim_{h\to 0}\frac{3}{(3-4x)(3-4x-4h)}$

   (iii) Take the limit to obtain the final answer: $f'(x) =$ $\frac{3}{(3-4x)^2}$

4. For the function in problem 3, find the equation of the tangent line when $x = 1$. $y + 1 = 3(x-1)$

5. Find the derivative of the following functions:

   (a) $f(x) = \cos^2 x \Rightarrow f'(x) =$ $-2\cos x \sin x$ or $-\sin 2x$ (b) $y = \sin x^2 \Rightarrow y' =$ $2x\cos x^2$

   (c) $y = \frac{x\cos x}{x+2} \Rightarrow \frac{dy}{dx} =$ $\frac{2\cos x - x(x+2)\sin x}{(x+2)^2}$ (d) $p = 4x^5\cos x - 2\cos x \Rightarrow p' =$ $20x^4\cos x - (4x^5-2)\sin x$

   (e) $g(t) = (2t+1)^{\frac{2}{3}}(t^2-1)^4 \Rightarrow \frac{dg}{dt} =$ $\frac{4}{3}(2t+1)^{-1/3}(t^2-1)^3[13t^2+6t-1]$

   (f) $\frac{d}{dx}\cos(\sin(\tan x^3)) =$ $-3x^2\sec^2 x^3 \cos(\tan x^3)\sin(\sin(\tan x^3))$

   (g) $\frac{d}{dx}\frac{4x^5-3x^3+\csc 1}{\pi x^3} =$ $\frac{8}{\pi}x - \frac{3\csc 1}{\pi}x^{-4}$ ($\frac{4}{\pi}x^2 - \frac{3}{\pi} + \frac{\csc 1}{\pi}x^{-3}$)

   (h) $\frac{d}{dx}\sqrt[4]{\frac{5x^{-3}+1}{x^{-3}}} =$ $\frac{1}{4}(5+x^3)^{-3/4}\cdot 3x^2 = \frac{3}{4}x^2(5+x^3)^{-3/4}$ ($(5+x^3)^{1/4}$)

Bonus:

(a) Find $\frac{dx}{dy}$ if $3xy + x^3y + xy^2 = 1$. $\frac{dx}{dy} =$ $\frac{-(3x + x^3 + 2xy)}{3y + y^2 + 3x^2y}$

(b) State the formula for the linear approximation of a function $f(x)$ at a point where $x = a$; i.e. what is the linearization of f at a?:

$L(x) =$ $f(a) + f'(a)(x-a)$