Math 201 Quiz 6A

October 14, 2014

Name: ANSWERS

Instructions: No calculators. Use your own scrap. Write your fully simplified answers in the space provided. Assume all given functions ae differentiable.

1. Complete the following formulas (you may write f to mean $f(x)$ and g to mean $g(x)$):

   (a) $\frac{d}{dx}x^n = nx^{n-1}$ (b) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'g - fg'}{g^2}$ (c) $\frac{d}{dx}(f(x)\cdot g(x)) = f'g + fg'$

   (d) $\frac{d}{dx}f(g(x)) = f'(g(x))\cdot g'(x)$ (e) $\frac{d}{dx}\tan x = \sec^2 x$ (f) $\frac{d}{dx}\csc x = -\csc x \cot x$

2. Use limits to define the derivative of a function $f(x)$: $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ or $\lim_{x\to a}\frac{f(x)-f(a)}{x-a} = f'(a)$

3. Use problem 2 to find the derivative of $f(x) = \frac{3x}{1-2x}$ by completing the following:

   (i) Set up the limit: $f'(x) = \lim_{h\to 0}\frac{\frac{3(x+h)}{1-2(x+h)} - \frac{3x}{1-2x}}{h}$ or $\lim_{x\to a}\frac{\frac{3x}{1-2x} - \frac{3a}{1-2a}}{x-a}$ $f'(a)$

   (ii) Fully simplify the limit (what you would have just before taking the limit): $\lim_{h\to 0}\frac{3}{(1-2x)(1-2x-2h)}$

   (iii) Take the limit to obtain the final answer: $f'(x) = \frac{3}{(1-2x)^2}$

4. For the function in problem 3, find the equation of the tangent line when $x = 1$. $y+3 = 3(x-1)$

5. Find the derivative of the following functions:

   (a) $f(x) = \sin^2 x \Rightarrow f'(x) = 2\sin x\cos x$ or $\sin 2x$ (b) $y = \cos x^2 \Rightarrow y' = -2x\sin x^2$

   (c) $y = \frac{x\sin x}{x+1} \Rightarrow \frac{dy}{dx} = \frac{\sin x + x(x+1)\cos x}{(x+1)^2}$ (d) $p = 5x^3\sin x - 2\sin x \Rightarrow p' = 15x^2\sin x + (5x^3-2)\cos x$

   (e) $g(t) = (t+1)^{\frac{2}{3}}(2t^2-1)^3 \Rightarrow \frac{dg}{dt} = \frac{2}{3}(t+1)^{-1/3}(2t^2-1)^2\left[20t^2+18t-1\right]$

   (f) $\frac{d}{dx}\sin(\cos(\tan x^2)) = -2x\sec^2 x^2 \sin(\tan x^2)\cos(\cos(\tan x^2))$

   (g) $\frac{d}{dx}\frac{2x^3-4x^2+\sec 1}{\pi x^2} = \frac{2}{\pi} - \frac{2\sec 1}{\pi}x^{-3}$

   $\frac{2}{\pi}x - \frac{4}{\pi} + \frac{\sec 1}{\pi}x^{-2}$

   (h) $\frac{d}{dx}\sqrt[5]{\frac{2x^{-3}+1}{x^{-3}}} = \frac{1}{5}(2+x^3)^{-4/5}\cdot 3x^2 = \frac{3}{5}x^2(2+x^3)^{-4/5}$

   $(2+x^3)^{1/5}$

Bonus:

(a) Find $\frac{dx}{dy}$ if $2xy + x^2y + xy^2 = 3$. $\frac{dx}{dy} = -\frac{(2x+x^2+2xy)}{2y+y^2+2xy}$

(b) State the formula for the linear approximation of a function $f(x)$ at a point where $x = a$; i.e. what is the linearization of f at a?:

$L(x) = f(a) + f'(a)(x-a)$