Math 201 Quiz 5B

September 19, 2014

Name: ANSWERS

Instructions: No calculators. Use your own scrap. Write your fully simplified answers in the space provided.

1. Compute the following limits, or write "DNE" if they do not exist. ∞ and $-\infty$ are valid answers:

   (a) $\lim_{x\to 2^-} \frac{x-4}{x^2-4x+4} =$ $-\infty$

   (b) $\lim_{x\to -\infty} \frac{-5x}{\sqrt{9+x^2}} =$ 5

   (c) $\lim_{x\to\infty} \frac{\cos^2 x}{x} =$ 0

   (d) $\lim_{x\to\infty} \frac{\cos x - x}{4x+\cos^2 x} =$ $-1/4$

   (e) $\lim_{x\to\infty} \left(\sqrt{5x^2+4x+7} - \sqrt{5x^2+x+3}\right) =$ $3/2\sqrt{5}$

   (f) $\lim_{t\to\infty} \frac{(2t^3+1)^2}{(2t+1)^4(t^2+t)} =$ $1/4$

   (g) $\lim_{x\to -\infty} \frac{\pi x^4 - 2x^3 + 15}{\cos(2) + 3x^4 - 7x} =$ $\pi/3$

   (h) $\lim_{x\to -\infty} \frac{4x^2+9x^4}{5-3x^3} =$ ∞

   (i) $\lim_{x\to -\infty} \frac{\pi x^3 + 3x^2 - 1}{\sin\left(\frac{2\pi}{7}\right)x^3 + 2x^4 - 3\pi} =$ 0

   (j) $\lim_{x\to -\infty} (x^4 - x^3) =$ ∞

2. Note that for the function $f(x) = 3/(x-2)$ we have that $f(-3) < 0$ and $f(3) > 0$. Since we have $f(-3) < 0 < f(3)$, are we guaranteed to have a solution to $f(x) = 0$ in the interval $(-3,3)$? If yes, say so and state what theorem you used. If no, state so and say why.

   No! The function is not continuous on $[-3,3]$; also, it is never zero! (non-zero numerator!)

3. State where the given functions are continuous. Use interval notation.

   (a) $f(x) = \begin{cases} 2x+1, x<0 \\ \cos x, 0 < x \le \pi/2 \\ x-\pi, x > \pi/2 \end{cases}$ $(-\infty, \pi/2) \cup (\pi/2, \infty)$

   (b) $f(x) = \frac{4}{\sqrt{1+7/x}}$ $(-\infty,-7)\cup(0,\infty)$.

4. Find the values of a and b that make the function continuous for all x.

   $f(x) = \begin{cases} x^2, x < 2 \\ ax+b, 2 \le x \le 5 \\ 3x+1, 5 < x \end{cases}$ For continuity, $a =$ 4 and $b =$ -4

Bonus:

(a) $\lim_{x\to 0} \frac{1-\cos x^2}{x^4} =$ $1/2$

(b) $\lim_{x\to\infty} \frac{2x^4 \cos x}{4-x^4} =$ DNE

(c) Let $f(x)$ be a differentiable function. Define $f'(x) =$ $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ or $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

(d) What is the equation of the tangent line to $y = x^4$ at the point where $x = 1$? $y-1 = 4(x-1)$ or $y = 4x-3$