Math 201 Quiz 5A

September 19, 2014

Name: ANSWERS

Instructions: No calculators. Use your own scrap. Write your fully simplified answers in the space provided.

1. Suppose $f(x) = \frac{2}{x-1}$. Note that $f(-2) < 0$ and $f(2) > 0$. As $f(-2) < 0 < f(2)$ are we guaranteed to have a root in the interval $(-2,2)$? If yes, say so and state what theorem you used. If no, state so and say why. (Recall, a root is a value c such that $f(c) = 0$.)

No! The function is not continuous on $[-2,2]$; also, it is never zero! (non-zero numerator!)

2. Compute the following limits, or write "DNE" if they do not exist. ∞ and $-\infty$ are valid answers:

(a) $\lim_{x\to\infty} \frac{\sin^2 x}{x} =$ 0 (b) $\lim_{x\to\infty} \frac{\sin x - x}{\sin^2 x + 3x} =$ $-1/3$

(c) $\lim_{x\to\infty} \left(\sqrt{3x^2+8x+6} - \sqrt{3x^2+3x+1}\right) =$ $5/2\sqrt{3}$

(d) $\lim_{x\to 2^+} \frac{x-4}{x^2-4x+4} =$ $-\infty$ (e) $\lim_{x\to-\infty} \frac{-3x}{\sqrt{4+x^2}} =$ 3

(f) $\lim_{t\to\infty} \frac{(2t^2+1)^2}{(t+1)^2(t^2+t)} =$ 4 (g) $\lim_{x\to-\infty} \frac{2-3x+\pi x^3}{\cos(2)+3x^4-7x} =$ 0

(h) $\lim_{x\to-\infty} \frac{4x^2+9x^3}{5-3x^3} =$ -3 (i) $\lim_{x\to-\infty} \frac{\pi x^7+3x^2-1}{\sin\left(\frac{2\pi}{7}\right)x^3+2x^4-3\pi} =$ $-\infty$

(j) $\lim_{x\to-\infty} (x^3 - x) =$ $-\infty$

3. State where the given functions are continuous. Use interval notation.

(a) $f(x) = \begin{cases} 2x, x < 0 \\ \sin x, 0 < x \le \pi \\ x - \pi, x > \pi \end{cases}$ $(-\infty, \infty)$

(b) $f(x) = \frac{3}{\sqrt{1+4/x}}$ $(-\infty, -4) \cup (0, \infty)$

4. Find the values of a and b that make the function continuous for all x.

$f(x) = \begin{cases} 2x^2 + 5, x < -1 \\ ax + b, -1 \le x \le 2 \\ 8x, x > 2 \end{cases}$ For continuity, $a =$ 3 and $b =$ 10

Bonus:

(a) $\lim_{x\to 0} \frac{1-\cos x^2}{x^4} =$ $1/2$

(b) Let $f(x)$ be a differentiable function. Define $f'(x) =$ $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ or $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

(c) What is the equation of the tangent line to $y = x^3$ at the point where $x = 1$? $y - 1 = 3(x-1)$ or $y = 3x - 2$

(d) $\lim_{x\to\infty} \frac{2x^2 \sin x}{x^2+4} =$ DNE