Related Rates Handout

For Math 201 Fall 2014

Related rates, as the name suggests, refers to relating the rates of change of several related quantities. As is often the case, the goal is to figure out the rate of change of one quantity if the rates of change of the other quantities are known (or can be known in theory, with *relative* ease—no pun intended, despite what the italicized text might suggest). The method of related rates is an application of implicit differentiation, which, as we've seen is an application of the chain rule. Another thing suggested by "rate" is the fact that the independent variable usually represents time, and is often denoted t. Let's get into the method and then do some examples!

<u>The Method of Related Rates</u>

1. **Read the problem carefully!** Read it again.
2. **If possible/necessary, draw a diagram. Label the quantities that are changing with variables and the quantities that are not changing by their constant values.** Think of the variables as functions of time.
3. **Write down the given information** in regards to the values of any rates that are known. Also **write down what you want to find**, and with what conditions. This will maintain your focus, grasshopper, as well as have the known values explicitly written down ready for use.
4. **Set up an equation that relates all the quantities under consideration.** If you drew a diagram, it will usually come in handy here. For instance, if your diagram is a triangle, the equation you come up with could be Pythogoras' theorem, the law of sines or cosines, or an application of SOHCAHTOA, etc. So the diagram would suggest what equation you would set up. Remember your geometry!
5. **Differentiate the equation in step 4 implicitly with respect to time**.
6. **Plug in your knowns and solve for the unknown that you seek.** At first, you may end up with several unknowns. In this case, you should be able to go back to the equation in step 4 to solve for all the unknowns you need in order to solve for the particular unknown you care about. Sometimes, you can also use the geometry suggested by the diagram to eliminate some of the variables in play to make your life easier.

Problems

1. A right circular cylinder undergoes expansion. Its radius is increasing at a rate of 1 m/s while its height is increasing at a rate of 2 m/s. At what rate is its volume changing when the radius is 1m in length and the height is 3m?

2. (a) A particle is moving along the curve $y = x^3 + 1$ in such a way that its y-coordinate is increasing at a rate of 2 units/sec. At what rate is its x-coordinate changing when $y = 9$?
(b) At this instant, how fast is the distance between the particle and the point (2,10) changing? Is the particle approaching (2,10) or getting farther from it at this point?

3. (a) A 15-foot ladder was resting against a vertical wall. Suddenly, and without warning, the foot of the ladder begins to slide away from the wall at a rate of 2 ft/s. At what rate is the top of the ladder sliding down the wall when the foot of the ladder is 9 feet from the wall?
(b) At what rate is the angle between the top of the ladder and the wall changing at the same instant? Is the angle increasing or decreasing?

4. At noon, ship A is 80 miles west of ship B. At this time, both ships set sail. Ship A sails south at a speed of 20 mph, while ship B sails north at a speed of 10 mph. How fast is the distance between the two ships changing at 2pm if both ships maintain constant speed?

5. A water tank has the shape of an inverted circular cone and has base radius 3 meters and height 9 meters. It starts out filled with water, but the water is leaking out from the bottom at a rate of 2 m^3/minute. For some odd reason, we want to know how the radius of the remaining water is decreasing when the water-radius is 1 meter. Please determine this rate, so that we may put our minds at ease. Recall that the volume of a circular cone is given by $V = \frac{1}{3}\pi r^2 h$.

6. A baseball diamond is a square with side 90 ft. A batter hits the ball and runs toward first base with a speed of 24 ft/s.
   (a) At what rate is his distance from second base decreasing when he is halfway to first base?
   (b) At what rate is his distance from third base increasing at the same moment?

P.S. There are many more interesting related rates problems in the text, and a relatively skimpy amount has been assigned for homework. I reserve the right to ask you any of the problems in this section of the text. Or any other calculus text for that matter, at the level of calc 1, of course. The situations in Related Rates can vary widely, so I suggest you practice more than is required in the homework.