Math 201 Quiz 7B

October 7, 2019

Name: ANSWERS

Instructions: No calculators. Use your own scrap. Write your fully simplified answers in the space provided.

1. Complete the following rules/computations:

   (a) $\frac{d}{dx}(f(x)-g(x)) =$ $f'(x)-g'(x)$ (b) $\frac{d}{dx}\sin x =$ $\cos x$

   (c) $\frac{d}{dx}f(g(x)) =$ $f'(g(x))\cdot g'(x)$ (d) $\frac{d}{dx}x^2 =$ $2x$

   (e) $\frac{d}{dx}\sqrt{x} =$ $\frac{1}{2\sqrt{x}}$ or $\frac{1}{2}x^{-1/2}$ (f) $\frac{d}{dx}\frac{1}{x} =$ $-1/x^2$

2. Use the limit definition of the derivative to find $f'(x)$ if $f(x)=\cos x$. Show your work below.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}$$

$$=\lim_{h\to 0}\frac{\cos x\cos h-\sin x\sin h-\cos x}{h}$$

$$=\lim_{h\to 0}\left[\cos x\,\frac{(\cos h-1)}{h}-\sin x\,\frac{(\sin h)}{h}\right]$$

($\frac{(\cos h-1)}{h}\to 0$, $\frac{(\sin h)}{h}\to 1$)

$$=-\sin x$$

3. Hence, by the above, find the equation of the tangent line to $f(x)$ at the point where $x=\frac{3\pi}{4}$.

$y =$ $-\frac{\sqrt{2}}{2}x+\frac{3\sqrt{2}\pi}{8}-\frac{\sqrt{2}}{2}$ OR $y=-\frac{\sqrt{2}}{2}x+\frac{\sqrt{2}}{8}(3\pi-4)$

4. Differentiate:

   (a) $\frac{d}{dx}\frac{3x+\frac{1}{x}+4}{5} =$ $\frac{3}{5}-\frac{1}{5x^2}$ (b) $\frac{d}{dx}\cos(\sqrt{x}-x^2+1) =$ $-\left(\frac{1}{2\sqrt{x}}-2x\right)\sin\left(\sqrt{x}-x^2+1\right)$

Bonus (can only be completed if all above are attempted): $\frac{-2x\sin(x^3)-3x^2\cos(x^3)(1-x^2)}{\sin^2(x^3)}$

1. Compute: $\frac{d}{dx}\frac{1-x^2}{\sin(x^3)} =$

2. State the power rule: $\frac{d}{dx}x^n=nx^{n-1}$

3. State the product rule: $\frac{d}{dx}(f\cdot g)=f'\cdot g+fg'$

4. State the quotient rule: $\frac{d}{dx}\left(\frac{f}{g}\right)=\frac{f'\cdot g-f\cdot g'}{g^2}$