Math 201 Quiz 7A

October 7, 2019

Name: ANSWERS

Instructions: No calculators. Use your own scrap. Write your fully simplified answers in the space provided.

1. Use the limit definition of the derivative to find $f'(x)$ if $f(x) = \cos x$. Show your work below.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\cos(x+h)-\cos x}{h}$$

$$= \lim_{h\to 0} \frac{\cos x\cos h - \sin x\sin h - \cos x}{h}$$

$$= \lim_{h\to 0} \left[\cos x \frac{(\cos h - 1)}{h} - \sin x \frac{(\sin h)}{h}\right] = -\sin x$$

($\frac{(\cos h-1)}{h} \to 0$, $\frac{(\sin h)}{h} \to 1$)

2. Hence, by the above, find the equation of the tangent line to $f(x)$ at the point where $x = \frac{\pi}{4}$.

$y =$ $-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}\pi}{8} + \frac{\sqrt{2}}{2}$ OR $y = -\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{8}(\pi+4)$

3. Complete the following rules/computations:

(a) $\frac{d}{dx}f(g(x)) =$ $f'(g(x))\cdot g'(x)$ (b) $\frac{d}{dx}(f(x)+g(x)) =$ $f'(x)+g'(x)$

(c) $\frac{d}{dx}\frac{1}{x} =$ $-1/x^2$ (d) $\frac{d}{dx}\sin x =$ $\cos x$

(e) $\frac{d}{dx}\sqrt{x} =$ $1/2\sqrt{x}$ or $\frac{1}{2}x^{-1/2}$ (f) $\frac{d}{dx}x^2 =$ $2x$

4. Differentiate:

(a) $\frac{d}{dx}\frac{x^2+\frac{1}{x}+5}{3} =$ $\frac{2x}{3} - \frac{1}{3x^2}$ (b) $\frac{d}{dx}\cos(5+x-\sqrt{x}) =$ $-\left(1-\frac{1}{2\sqrt{x}}\right)\sin(5+x-\sqrt{x})$

Bonus (can only be completed if all above are attempted):

1. State the product rule: $\frac{d}{dx}(f\cdot g) = f'\cdot g + f\cdot g'$
2. State the power rule: $\frac{d}{dx}x^n = nx^{n-1}$
3. State the quotient rule: $\frac{d}{dx}(f/g) = \frac{f'g - fg'}{g^2}$
4. Compute: $\frac{d}{dx}\frac{x^2+1}{\sin(x^2)} =$ $\frac{2x\sin(x^2) - 2x\cos(x^2)(x^2+1)}{\sin^2(x^2)}$