Math 201 Quiz 6B

October 2, 2019

Name: ANSWERS

Instructions: No calculators. Use your own scrap. Write your fully simplified answers in the space provided.

1. Suppose $f(x) = \frac{3x}{x-2}$. Note that $f(-1) > 0$ and $f(1) < 0$. As $f(1) < 0 < f(-1)$ are we guaranteed to have a root in the interval $(-1,1)$? If yes, say so and state what theorem you used. If no, state so and say why. (Recall, a root is a value c such that $f(c) = 0$.)

   Yes! By the Intermediate Value Theorem.

2. For the Intermediate Value Theorem to apply to a function $f(x)$ on the interval $[a, b]$, what assumption(s) must be made about $f(x)$?

   $f(x)$ is continuous on $[a,b]$.

3. State the ϵ-δ definition of what $\lim_{x\to a} f(x) = L$ means: $\forall \varepsilon > 0, \exists \delta > 0$ so that if $0 < |x-a| < \delta$, then $|f(x) - L| < \varepsilon$.

4. Use the ϵ-δ definition to prove $\lim_{x\to 2}(2x-1) = 3$.

   Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon/2$.
   Then, if $0 < |x-2| < \delta$, we have
   $|(2x-1)-3| = |2x-4| = 2|x-2| < 2\delta = 2\cdot\frac{\varepsilon}{2} = \varepsilon$. ∎

5. Using an equation, state the definition of the derivative of a function $f(x)$, assuming it exists.

   $$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

6. (a) Use limits to compute the derivative of $f(x) = 2 + \frac{1}{x}$. (Show your work below.)

   $$f'(x) = \lim_{h\to 0} \frac{2 + \frac{1}{x+h} - \left(2 + \frac{1}{x}\right)}{h}$$

   $$= \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \cdot \frac{x(x+h)}{x(x+h)}$$

   $$= \lim_{h\to 0} \frac{x - (x+h)}{hx(x+h)}$$

   $$= \lim_{h\to 0} \frac{-1}{x(x+h)}$$

   $$= -\frac{1}{x^2}$$

   (b) Hence, state the equation of the tangent line to $f(x)$ at the point where $x = 1$. $y = -x + 4$ or $y - 3 = -(x-1)$

Bonus:

1. State the chain rule: $\frac{d}{dx} f(g(x)) = f'(g(x))\cdot g'(x)$

2. Complete the rules:
   (a) $\frac{d}{dx}(cf(x)) =$ $c\,f'(x)$ (b) $\frac{d}{dx}(f(x) - g(x)) =$ $f'(x) - g'(x)$