Math 201 Quiz 6A

October 2, 2019

Name: ANSWERS

Instructions: No calculators. Use your own scrap. Write your fully simplified answers in the space provided.

1. For the Intermediate Value Theorem to apply to a function $f(x)$ on the interval $[a, b]$, what assumption(s) must be made about $f(x)$?

   $f(x)$ is continuous on $[a,b]$.

2. Suppose $f(x) = \frac{3}{x-2}$. Note that $f(-3) < 0$ and $f(3) > 0$. As $f(-3) < 0 < f(3)$ are we guaranteed to have a root in the interval $(-3,3)$? If yes, say so and state what theorem you used. If no, state so and say why. (Recall, a root is a value c such that $f(c) = 0$.)

   No! f is not continuous on $[-3,3]$.

3. State the ϵ-δ definition of what $\lim_{x\to a} f(x) = L$ means: $\forall \varepsilon > 0$, $\exists \delta > 0$ so that if $0 < |x-a| < \delta$, then $|f(x) - L| < \varepsilon$.

4. Use the ϵ-δ definition to prove $\lim_{x\to 3}(3x - 2) = 7$.

   Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon/3$.
   Then, if $0 < |x-3| < \delta$, we have
   $|(3x-2)-7| = |3x-9| = 3|x-3| < 3\delta = 3\cdot\frac{\varepsilon}{3} = \varepsilon.$ ■

5. Using an equation, state the definition of the derivative of a function $f(x)$, assuming it exists.

   $$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

6. (a) Use limits to compute the derivative of $f(x) = 3 - \frac{1}{x}$. (Show your work below.)

   $$f'(x) = \lim_{h\to 0} \frac{3 - \frac{1}{x+h} - (3 - \frac{1}{x})}{h}$$
   $$= \lim_{h\to 0} \frac{\frac{1}{x} - \frac{1}{x+h}}{h} \cdot \frac{x(x+h)}{x(x+h)}$$
   $$= \lim_{h\to 0} \frac{(x+h) - x}{hx(x+h)}$$
   $$= \lim_{h\to 0} \frac{1}{x(x+h)}$$
   $$= \frac{1}{x^2}$$

   (b) Hence, state the equation of the tangent line to $f(x)$ at the point where $x = 1$. $y = x+1$

   OR $y - 2 = 1(x-1)$

Bonus:

1. State the chain rule: $\frac{d}{dx} f(g(x)) = f'(g(x))\cdot g'(x)$

2. Complete the rules:
   (a) $\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$ (b) $\frac{d}{dx}(cf(x)) = c f'(x)$