

TEST 3A

1/ Using $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$
with $(x_1, y_1) = (-1, 3)$, $(x_2, y_2) = (5, 1)$
we have $M = \left(\frac{-1+5}{2}, \frac{3+1}{2} \right)$
 $= (2, 2)$

2/ Using $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$
with $(x_1, y_1) = (-1, 3)$, $(x_2, y_2) = (5, 1)$
we have $d = \sqrt{(5+1)^2 + (1-3)^2}$
 $= 2\sqrt{10}$

3/ $D = \begin{vmatrix} 1 & 1 & 1 \\ -1 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 2$

$$D_x = \begin{vmatrix} 6 & 1 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 2$$

so that $x = \frac{D_x}{D} = \frac{2}{2} = 1$

4/ $x + 2y = 5$ — ①
 $x^2 + y^2 = 25$ — ②

From ①: $x = 5 - 2y$

Plug this in ② to obtain

$$(5-2y)^2 + y^2 = 25$$

$$\Rightarrow y = 0 \text{ or } y = 4$$

Plug each of these in ① to get

$$x = 5 \text{ or } x = -3, \text{ respectively.}$$

\therefore the solutions are $(5, 0)$, $(-3, 4)$

5/ $12 - 3y^2 + 6x = 3x^2 - 12y$

$$\Rightarrow 4 = x^2 - 2x + y^2 - 4y$$

$$\Rightarrow (1)^2 + (-2)^2 + 4 = x^2 - 2x + (-1)^2 + y^2 - 4y + (-2)^2$$

$$\Rightarrow 9 = (x-1)^2 + (y-2)^2$$

\therefore center: $(1, 2)$, radius = 3

6/ center = $(-1, 2)$, point = $(1, 2)$
plug these in $(x-h)^2 + (y-k)^2 = r^2$
 $\Rightarrow (1+1)^2 + (2-2)^2 = r^2$
 $\Rightarrow r^2 = 4$
 \therefore the equation is: $(x+1)^2 + (y-2)^2 = 4$

7/ $y = x^2 - 4x - 5 = (x-5)(x+1)$

For x-int: set $y = 0$

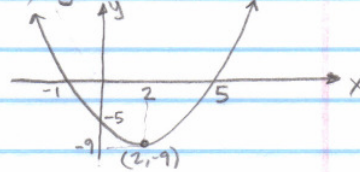
$$\Rightarrow (x-5)(x+1) = 0 \Rightarrow x = 5 \text{ or } x = -1$$

For y-int: set $x = 0$

$$\Rightarrow y = -5$$

For vertex: $x = \frac{-b}{2a} = \frac{4}{2} = 2 \Rightarrow (2, -9)$

when $x = 2$, $y = 2^2 - 4(2) - 5 = -9$



8/ $2x^2 + 6x + 3 = 0$

Using the quadratic formula,

$$x = \frac{-6 \pm \sqrt{(-6)^2 - 4(2)(3)}}{2(2)} = \frac{-3 \pm \sqrt{3}}{2}$$

9/ By problem 1, the center = $M = (2, 2)$

By problem 2, the radius = $\frac{1}{2}d = \sqrt{10}$

\therefore the equation is: $(x-2)^2 + (y-2)^2 = 10$

10/ $y = 3x - x^2 = x(3-x)$

\Rightarrow x-int: $x = 0, x = 3$; y-int: $y = 0$

vertex: $x = \frac{-3}{2} = \frac{3}{2} \Rightarrow y = \frac{9}{4} \Rightarrow \left(\frac{3}{2}, \frac{9}{4} \right)$

