MATH 190 - Diagnostic Exam. Answer all questions

Name: JHEVON SMITH

1. Solve the formula for the specified variable: $A = P + P\,rt$ for P.

$$P = \frac{A}{1+rt}$$

2. Let f(x)=5x+3 and g(x)= $2x^2 - 4$ find f[g(x)]

$$f(g(x)) = 5(g(x)) + 3$$
$$= 5(2x^2 - 4) + 3$$
$$= 10x^2 - 17$$

3. Find the slope of the line containing the pair of points: (5, 2) and (-3, 7)

$$m = \frac{7-2}{-3-5} = \frac{5}{-8} = -\frac{5}{8}$$

4. Which one of the following is an equation of a line that passes through the point $(2,-3)$ with a slope of 4?

[A] $y = 4x + 11$ [B] $y = 4x - 5$ [C] $y = \frac{1}{4}x - 3$ [D] $y = 4x - 11$ [E] None of these

5. If $g(x) = -x^2 + 3$ then find $g(-4)$.

$$g(-4) = -(-4)^2 + 3$$
$$= -13$$

6. If $f(x) = x^2 + 2x$, then find $f(a+h) - f(a)$.

$$f(a+h) - f(a) = (a+h)^2 + 2(a+h) - (a^2 + 2a)$$
$$= a^2 + 2ah + h^2 + 2a + 2h - a^2 - 2a$$
$$= 2ah + h^2 + 2h$$

7. Find the coordinates of the vertex of the parabola: $y = 2x^2 + 4x - 3$

$$x = -\frac{b}{2a} = -\frac{4}{2(2)} = -1$$

$$\Rightarrow y = 2(-1)^2 + 4(-1) - 3 \Rightarrow \text{Vertex } (-1, -5)$$
$$= 2 - 4 - 3$$
$$= -5$$

8. Write the equation of the line that passes through P (-3, 10) and is perpendicular to 4x+y=6.

$\hookrightarrow m_0 = -4$

$$\Rightarrow m_1 = \frac{1}{4}$$

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 10 = \frac{1}{4}(x + 3)$$

9. Simplify: $x^3\sqrt{12x^9y^{17}} + y^5\sqrt{75x^{15}y^7}$

$= x^3\sqrt{4x^8y^{16}}\sqrt{3xy} + y^5\sqrt{25x^{14}y^6}\sqrt{3xy}$

$= x^3 \cdot 2x^4y^8\sqrt{3xy} + y^5 \cdot 5x^7y^3\sqrt{3xy}$

$= 2x^7y^8\sqrt{3xy} + 5x^7y^8\sqrt{3xy}$

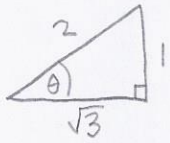
$= 7x^7y^8\sqrt{3xy}$

10. Which of the following identities is equal to 1?

A) $\sin^2\theta - \cos^2\theta$ B) $\csc^2\theta + \cot^2\theta$ C) $\sec^2\theta + \tan^2\theta$ D) $\csc^2\theta - \cot^2\theta$

E) none of these

11. If $\sin\theta = \frac{1}{2}$, and θ is in quadrant II then $\sec\theta =$



$\sec\theta = -\frac{1}{\cos\theta}$ (negative since θ is in quad. II).

$= -\frac{hyp}{adj}$

$= -\frac{2}{\sqrt{3}}$

12. Find the value of c that completes the perfect square $x^2 + 38x + c$

$c = \left(\frac{38}{2}\right)^2 = 19^2$

13. Divide and simplify: $\frac{x^2+x-6}{x^2+6x+9} \div \frac{x^2-4}{x^2-9} =$

$\frac{(x+3)(x-2)}{(x+3)^2} \cdot \frac{(x-3)(x+3)}{(x-2)(x+2)}$

$= \frac{x-3}{x+2}$

14. Solve the system: $2x - 3y = -4$ — ①
$x + 4y = 9$ — ②

$2x - 3y = -4$ — ①
$-2x - 8y = -18$ — ② × −2
$-11y = -22$
$y = 2$

$\Rightarrow 2x - 3(2) = -4$
$2x = 2$
$x = 1$

Soln: $(x, y) = (1, 2)$

15. $(\csc^2\theta - 1)\tan^2\theta = \cot^2\theta \cdot \tan^2\theta$

a. 1 b. $1 - \tan^2\theta$ c. $\sec^2\theta - 1$ d. $\csc^2\theta \sec^2\theta + 1$ e. $\sec^4\theta + 1$

f. None of these

16. Simplify $\dfrac{(-8a^3b^2c)^2}{(4a^3bc^2)^3} = \dfrac{64a^6b^4c^2}{64a^9b^3c^6}$

$= a^{-3}bc^{-4}$

$= \dfrac{b}{a^3c^4}$

17. Expand $(\sqrt{7} - 3\sqrt{2})^2 = 7 - 6\sqrt{7}\sqrt{2} + 9(2)$

$= 25 - 6\sqrt{14}$

18. $\dfrac{x+1}{x^2-9x+14} - \dfrac{x}{2x-4} = \dfrac{2}{2}\dfrac{x+1}{(x-7)(x-2)} - \dfrac{x}{2(x-2)} \cdot \dfrac{x-7}{x-7}$

$= \dfrac{2(x+1) - x(x-7)}{2(x-2)(x-7)}$

$= \dfrac{2x+2-x^2+7x}{2(x-2)(x-7)}$

$= -\dfrac{(x^2-9x-2)}{2(x-2)(x-7)}$

19. Solve for x: $3x+5=\sqrt{2-2x}$

By plugging in

A)2 B)no solution C)1 (D)-1) E)none of these

20. $(5x^4-2x^2+1)\div(x+1)=$ $5x^3-5x^2+3x-3+\frac{4}{x+1}$

$$x+1\overline{)\,5x^4+0x^3-2x^2+0x+1}$$

$$5x^3-5x^2+3x-3$$

$$-(5x^4+5x^3)$$

$$-5x^3-2x^2$$

$$-(-5x^3-5x^2)$$

$$3x^2+0x$$

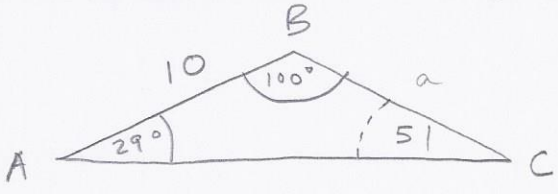
$$-(3x^2+3x)$$

$$-3x+1$$

$$-(-3x-3)$$

$$4$$

21. In triangle ABC, the measure of angle A is 29 degrees, angle B measures 100 degrees and the length of side AB is 10 cm. Find the length of BC, rounded off to two decimal places.

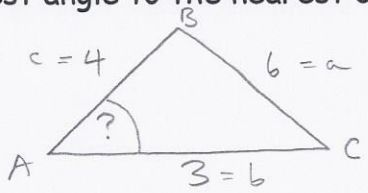


$$\frac{BC}{\sin 29}=\frac{10}{\sin 51}$$ by the law of sines

$$\Rightarrow BC=\frac{10\sin 29}{\sin 51}$$

$$=6.24$$

22. Find the largest angle to the nearest degree of a triangle whose sides measure 3, 4 and 6 inches.



$a^2 = b^2 + c^2 - 2bc\cos A$

$36 = 9 + 16 - 24\cos A$

$\Rightarrow \cos A = \frac{-11}{24}$

$\Rightarrow A = \cos^{-1}\left(-\frac{11}{24}\right)$

$\approx 117.28^\circ$

23. Find the center and the radius of the circle whose equation is $y^2 = -x^2 - 6y - 2x - 9$

$\Rightarrow x^2 + 2x + y^2 + 6y = -9$

$(x+1)^2 + (y+3)^2 = 1$

center $= (-1, -3)$

radius $= 1$

24. Use Cramer's Rule to solve for z: $\begin{cases} x - y + 2z = 5 \\ 2x + 3z = 11 \\ -x - 2y + 2z = 1 \end{cases}$

$$D = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 0 & 3 \\ -1 & -2 & 2 \end{vmatrix} = 1\begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix} + 2\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = (4+3) + 2(3-4)$$

$= 7 - 2$

$= 5$

$$D_z = \begin{vmatrix} 1 & -1 & 5 \\ 2 & 0 & 11 \\ -1 & -2 & 1 \end{vmatrix} = 1\begin{vmatrix} 2 & 11 \\ -1 & 1 \end{vmatrix} + 2\begin{vmatrix} 1 & 5 \\ 2 & 11 \end{vmatrix} = (2+11) + 2(11-10)$$

$= 13 + 2$

$= 15$

$\Rightarrow z = \frac{D_z}{D} = 3$

25. Solve for x and y: $\begin{cases} x+y=4 & -① \\ x^2-y=2 & -② \end{cases}$

ln ② $y = x^2 - 2$

$\Rightarrow$ ln ① $x + x^2 - 2 = 4$

$\Rightarrow x^2 + x - 6 = 0$

$\Rightarrow (x+3)(x-2) = 0$

$x = -3, \quad x = 2$

$\Rightarrow y = 7 \quad | \quad \Rightarrow y = 2$

$\Rightarrow$ Soln $(x,y) = (-3,7)$ and $(x,y) = (2,2)$

26. Given: (2,3) and (-5,4) find A)the midpoint B)the distance

A) $M = \left(\frac{2-5}{2}, \frac{3+4}{2}\right)$

$= \left(-\frac{3}{2}, \frac{7}{2}\right)$

B) $D = \sqrt{(2+5)^2 + (3-4)^2}$

$= \sqrt{49+1}$

$= \sqrt{50}$

$= 5\sqrt{2}$

27. Solve for x: $16x = 16 - \frac{13}{x}$

$\Rightarrow 16x^2 = 16x - 13$

$\Rightarrow 16x^2 - 16x + 13 = 0$

$\Rightarrow x = \frac{16 \pm \sqrt{16^2 - 4(16)(13)}}{2(16)}$

no real soln!