Math 392 Quiz 7B

March 20, 2018

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

In all formulas that require the use of a normal vector or its magnitude, don't just write $\vec{n}$ or $|\vec{n}|$, but rather the appropriate formula to give the normal vector or its magnitude.

1. Suppose a surface S_1 is given explicitly as $y = f(x,z)$. Give a formula to find a normal vector to S_1 at an arbitrary point on S_1. $\vec{n} =$ $\langle -f_x, 1, -f_z \rangle$ or $\langle f_x, -1, f_z \rangle$
2. Suppose a surface S_2 is parametrized by $\vec{r}(s,t)$. Give a formula to find a normal vector to S_2 at an arbitrary point on S_2. $\vec{n} =$ $\vec{r}_s \times \vec{r}_t$
3. For S_1 above, define $\iint_{S_1} f(x,y,z)dS =$ $\iint_D f(x, f(x,z), z)\sqrt{f_x^2 + f_z^2 + 1}\, dA$
4. For S_2 above, define $\iint_{S_2} f(x,y,z)dS =$ $\iint_D f(\vec{r}(s,t))\,|\vec{r}_s \times \vec{r}_t|\, dA$
5. Set up an integral to compute $\iint_S x\, dS$ where S is the part of the paraboloid $x = y^2 + z^2$ that lies inside the cylinder $y^2 + z^2 = 4$.

   Integral, fully set-up in coordinate system of your choice: $\int_0^{2\pi}\int_0^2 r^2\sqrt{4r^2+1}\; r\,dr\,d\theta$

Bonus:

1. Final answer for problem 5 above: $\frac{\pi}{8}\left(\frac{17^{5/2}}{5} - \frac{17^{3/2}}{3} + \frac{2}{15}\right)$
2. Suppose $\vec{F}$ is a 3D vector field whose components have continuous partial derivatives throughout $\mathbb{R}^3$. Define

   (a) $\iint_{S_1} \vec{F} \cdot d\vec{S} =$ $\iint_D \vec{F}(x, f(x,z), z) \cdot \langle -f_x, 1, -f_z \rangle\, dA$

   (b) $\iint_{S_2} \vec{F} \cdot d\vec{S} =$ $\iint_D \vec{F}(\vec{r}(s,t)) \cdot (\vec{r}_s \times \vec{r}_t)\, dA$

3. Let S be the boundary of the region bounded by the ~~paraboloid~~ cone $z = \sqrt{x^2+y^2}$ and the plane $z = 1$. Note that S is closed. Sketch S, along with positively oriented vector(s) on its surface.