Math 392 Quiz 7A

March 20, 2018

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

In all formulas that require the use of a normal vector or its magnitude, don't just write $\vec{n}$ or $|\vec{n}|$, but rather the appropriate formula to give the normal vector or its magnitude.

1. Suppose a surface S_1 is parametrized by $\vec{r}(u,v)$. Give a formula to find a normal vector to S_1 at an arbitrary point on S_1. $\vec{n} =$ $\vec{r}_u \times \vec{r}_v$

2. Suppose a surface S_2 is given explicitly as $x = f(y,z)$. Give a formula to find a normal vector to S_2 at an arbitrary point on S_2. $\vec{n} =$ $\langle 1, -f_y, -f_z \rangle$ or $\langle -1, f_y, f_z \rangle$

3. For S_1 above, define $\iint_{S_1} f(x,y,z)\,dS =$ $\iint_D f(\vec{r}(u,v))\,|\vec{r}_u \times \vec{r}_v|\,dA$

4. For S_2 above, define $\iint_{S_2} f(x,y,z)\,dS =$ $\iint_D f(f(y,z),y,z)\sqrt{f_y^2+f_z^2+1}\,dA$

   should have given this a different name... sorry!

5. Set up an integral to compute $\iint_S y\,dS$ where S is the part of the paraboloid $y = x^2+z^2$ that lies inside the cylinder $x^2+z^2 = 1$.

   Integral, fully set-up in coordinate system of your choice: $\int_0^{2\pi}\int_0^1 r^2\sqrt{4r^2+1}\; r\,dr\,d\theta$

Bonus:

1. Final answer for problem 5 above: $\frac{\pi}{8}\left(\frac{5^{5/2}}{5} - \frac{5^{3/2}}{3} + \frac{2}{15}\right)$

2. Suppose $\vec{F}$ is a 3D vector field whose components have continuous partial derivatives throughout $\mathbb{R}^3$. Define

   (a) $\iint_{S_1} \vec{F}\cdot d\vec{S} =$ $\iint_D \vec{F}(\vec{r}(u,v))\cdot(\vec{r}_u \times \vec{r}_v)\,dA$

   (b) $\iint_{S_2} \vec{F}\cdot d\vec{S} =$ $\iint_D \vec{F}(f(y,z),y,z)\cdot\langle 1, -f_y, -f_z\rangle\,dA$

3. Let S be the boundary of the region bounded by the paraboloid $z = x^2+y^2$ and the plane $z = 1$. Note that S is closed. Sketch S, along with positively oriented vector(s) on its surface.