Math 391 Mock Test 1
October 27, 2019

Name: _____

Note that both sides of each page may have printed material.

Instructions:

1. Read the instructions.
2. Panic!!! Kidding, don't panic! I repeat, do NOT panic!
3. Complete all problems in the exam. Problems in the bonus section are optional.
4. Show ALL your work to receive full credit. You will get 0 credit for simply writing down the answers (unless it's a case of fill in the blank or state a definition, etc.)
5. Write neatly so that I am able to follow your sequence of steps and box your answers. Problems with no indicated answer, or several contradictory answers will be considered incorrect.
6. Read through the exam and complete the problems that are easy (for you) first!
7. No scrap paper, calculators, notes or other outside aids allowed—including divine intervention, telepathy, knowledge osmosis, the smart kid that may be sitting beside you or that friend you might be thinking of texting.
8. In fact, **cell phones should be out of sight! Smart watches too.**
9. Use the correct notation and write what you mean! x^2 and $x2$ are not the same thing, for example, and I will grade accordingly.
10. Do NOT commit any of the blasphemies or mistakes I mentioned in the syllabus. I will actually mete out punishment in the way I said I would. I wasn't kidding.
11. Other than that, have fun and good luck!

Remember: math is fun, math is beautiful, this test is *not* hard, there is no spoon.

1. (5 points each)
Find the general solution of the following:

(i) $\frac{dy}{dx} = \frac{1-3x^2y-x^3y}{x^3}$, $x > 0$

(ii) $xdx + 2ye^{-x}dy = 0$

2. (20 points) Find the general solution of the following:

(a) $(9x^2 + 7xy + y^2)dx - x^2dy = 0$

(b) $(e^x \cos(xy) - ye^x \sin(xy) - 2x - y)dx - (xe^x \sin(xy) - 2y + x)dy = 0$

3. Marty was voted the class of 1985's "Most Likely to Abuse Time Travel"—a prediction that was realized shortly after he married his high school sweetheart. He goes back to the future and, unfortunately, discovers that he will die at age 75. Knowing that he will retire at age 55 (being the abuser of time that he is), he decides to go back to the present and save up enough money to make it to age 75 comfortably. When he retires at 55, he plans to put his savings into an account earning 10% interest, compounded continuously. He also plans to withdraw $80,000 per year to live on and enjoy his remaining days having adventures with Doc. Brown.

   (a) (4 points) Suppose $P(t)$ represents the balance of his account at time t (in years). Write down an equation for $\frac{dP}{dt}$.

   (b) (6 points) Solve the equation in part (a), assuming an initial investment of P_0 dollars.

4. A 200 gallon tank initially contains 100 gallons of water with a pound of salt dissolved in it. Water containing 1/5 lb of salt per gallon is pumped into the tank at a rate of 10 gallons per minute, and the mixture is pumped out of the tank at 5 gallons per minute. Suppose $Q(t)$ is the amount of salt in the tank at time t minutes.

   (a) (10 points) Set up a differential equation with initial condition to describe $Q(t)$ and solve it.

   (b) (5 points) What will be the concentration of salt in the tank at the point of overflow?

5. (5 points each) Solve the following ODEs:

(a) $6y'' - 7y' - 3y = 0$

(b) $4y'' + 12y' + 9y = 0$

(c) $y'' + 6y' + 11y = 0$

6. (10 points) Given the differential equation $2t^2y'' + ty' - y = 0, t > 0$; notice that $y_1 = t^{-1/2}$ is a solution. Use reduction of order to find a second, linearly independent solution, y_2. *Prove that your second solution is linearly independent to the first.*

7. (20 points) Solve the ODE: $y'' + y = \sec t$, $0 \leq t < \frac{\pi}{2}$, subject to $y(0) = 4, y'(0) = 9$.

Bonus Problems:

1. (5 points) Find the general solution of $y''' - 2y'' + 4y' - 8y = 0$

2. (5 points) Find the general solution of $y + (2x - ye^y)y' = 0$. Hint: a function $\mu(y)$ will turn this into an Exact Equation.

3. (5 points) Write down the form for the general solution to $y^{(6)} + 2y''' + y = t^2 + e^{-t} + 5e^{t/2} \sin\frac{\sqrt{3}t}{2}$. For your y_p, you need not find the arbitrary constants, but it must be minimal and have the fewest terms possible.