Math 346 Quiz 7
March 20, 2018

Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. Let $A = \begin{pmatrix} 3 & 2 & -1 \\ 2 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}$.

$$\begin{vmatrix} 3 & 2 & -1 \\ 2 & 2 & -1 \\ 0 & -1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 0 & -1 & 1 \end{vmatrix} \, R_1 - R_2 = 1 \begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix}$$

(a) Compute $\det A =$ 1 (You may consider using pivotal condensation to help)

(b) Find $A^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ -2 & 3 & 1 \\ -2 & 3 & 2 \end{pmatrix}$ (Write "N/A" if A^{-1} does not exist, and state your reason in the space below. Otherwise, show your work for how you computed the inverse.)

Row Reduction

$$\left(\begin{array}{ccc|ccc} 3 & 2 & -1 & 1 & 0 & 0 \\ 2 & 2 & -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 2 & 2 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 \end{array}\right) \begin{matrix} R_1 - R_2 \\ \\ R_3 \times -1 \end{matrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 2 & -1 & -2 & 3 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 \end{array}\right) R_2 - 2R_1$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -2 & 3 & 2 \end{array}\right) \begin{matrix} \\ R_3 \\ R_2 - 2R_3 \end{matrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -2 & 3 & 1 \\ 0 & 0 & 1 & -2 & 3 & 2 \end{array}\right) R_2 + R_3$$

OR

Adjoint Method

$$A^{-1} = \frac{1}{\det A} \operatorname{Adj}(A)$$

$$= \frac{1}{1} \begin{pmatrix} \begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 2 \\ 0 & -1 \end{vmatrix} \\ -\begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} & \begin{vmatrix} 3 & -1 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 3 & 2 \\ 0 & -1 \end{vmatrix} \\ \begin{vmatrix} 2 & -1 \\ 2 & -1 \end{vmatrix} & -\begin{vmatrix} 3 & -1 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} 3 & 2 \\ 2 & 2 \end{vmatrix} \end{pmatrix}^T$$

$$= \begin{pmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & 1 & 2 \end{pmatrix}^T$$

$$= \begin{pmatrix} 1 & -1 & 0 \\ -2 & 3 & 1 \\ -2 & 3 & 2 \end{pmatrix}$$

(c) Solve $\begin{array}{rcrcrcl} 3x & + & 2y & - & z & = & 1 \\ 2x & + & 2y & - & z & = & 1. \\ & & -y & + & z & = & 0 \end{array}$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$ $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \leftarrow A^{-1}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ (Write "inconsistent" if no solution exists)

2. Define, via an equation, what it means for the matrix A to be *symmetric*: $A = A^T$

Bonus:

1. Let V be a vector space over a scalar field $\mathcal{F}$. Let $\vec{u}, \vec{v} \in V$ and suppose $k, l \in \mathcal{F}$. Which of the following are not a property of V? Circle the appropriate one(s).

- $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$
- $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
- $(k + l)\vec{u} = k\vec{u} + l\vec{u}$
- Every $\vec{u}$ has a negative.
- $1\vec{u} = \vec{u}$
- For each $\vec{u} \in V$, if $\vec{u} \neq \vec{0}$, then there exists a vector $\vec{u}^{-1}$ such that $\vec{u} \cdot \vec{u}^{-1} = 1$. (Remember, 1 is the multiplicative identity.)

2. Let $U \subseteq V$, where V is a vector space over a field $\mathcal{F}$. What two conditions must U fulfill to be a subspace of V?

(i) U is closed under addition.

(ii) U is closed under scalar multiplication.