Math 308 - Summer 2018

Selected Solutions to HW set 7

Problems 30, 34 and 64 were graded for HW 7.

Disclaimer: If you have questions about any of the other problems, see me in office hours. Consider all problems important, not just the ones I provide solutions for. Also consider it important to do *more* than what is required for homework. Also note there are many ways to prove statements in general, so my proof might not look like yours, and that's fine as long as yours is correct :p

Prove or disprove:

30. Let $x \in \mathbb{Z}$. If $4x + 7$ is odd, then x is even.

This is false.

Since $4x + 7$ is always odd (it is the sum of an odd integer and an even integer), knowing that it is odd does not imply a particular parity of x. As a counter example, consider $x = 1$. Then $4x + 7 = 11$ which is odd. However, $x = 1$ is not even. Thus we have a true statement implying a false statement.

34. For every two sets A and B, $(A \cup B) - B = A$.

This is false.

One may use a typical Venn diagram to realize something is fishy here. To take an extreme counter-example:

Let $A = B = \{1\}$. Then $(A \cup B) - B = \emptyset$, but A is non-empty.

There are many ways to create a counter-example here. For instance, take any non-empty set A and choose B to be any set whose intersection with A is non-empty. Then the statement will be false; because subtracting B will subtract elements of A, and so you won't be left with the set A in the end.

64. There exists an irrational number a and a rational number b such that a^b is irrational.

This is true.

Pf: Choose $b = 1 \in \mathbb{Q}$, and let a be any irrational number. Then $a^b = a^1 = a$, which is irrational. ■