**Math 308 - Summer 2018**

**Selected Solutions to HW set 2**

Problems 16, 22, and 46 were graded for HW 2.

Disclaimer: If you have questions about any of the other problems, see me in office hours. Consider all problems important, not just the ones I provide solutions for. Also consider it important to do *more* than what is required for homework.

16. For sets $A = \{1,2,\dots,10\}$ and $B = \{2,4,6,9,12,25\}$, consider the statements

$$P: A \subseteq B. \qquad Q: |A - B| = 6.$$

Determine which of the following statements are true.

First note that $A$ is NOT a subset of $B$, since, for instance, $1 \in A$ but $1 \notin B$. So the statement $P$ is false.

Also, $A - B = \{1,3,5,7,8,10\} \Longrightarrow |A - B| = 6$. So statement $Q$ is true.

All the following answers can be verified by looking at the appropriate row of a truth table involving the statements, but we will neglect to show the truth tables here. Our choice is justified by he statements in the answers.

(a) $P \vee Q$
This is TRUE since $Q$ is true.

(b) $P \vee (\sim Q)$
This is FALSE since both $P$ and $\sim Q$ are false.

(c) $P \wedge Q$
This is FALSE since $P$ is false.

(d) $(\sim P) \wedge Q$
This is TRUE since both $\sim P$ and $Q$ are true.

(e) $(\sim P) \vee (\sim Q)$
This is TRUE since $\sim P$ is true.

22. Consider the statements:

$$P: \sqrt{2} \text{ is rational.} \qquad Q: \frac{2}{3} \text{ is rational.} \qquad R: \sqrt{3} \text{ is rational.}$$

Write each of the following in words and indicate whether the statement is true or false.

Note that $P$ is false, $Q$ is true and $R$ is false.

(a) $(P \wedge Q) \Rightarrow R$
In words: If $\sqrt{2}$ is rational and $\frac{2}{3}$ is rational, then $\sqrt{3}$ is rational.
This is TRUE since $P \wedge Q$ is false ($P \wedge Q$ is false since $P$ is false).

(b) $(P \wedge Q) \Rightarrow (\sim R)$

In words: If $\sqrt{2}$ is rational and $\frac{2}{3}$ is rational, then $\sqrt{3}$ is not rational.

This is TRUE for exactly the same reason as in part (a).

(c) $\big((\sim P) \wedge Q\big) \Rightarrow R$

In words: If $\sqrt{2}$ is not rational and $\frac{2}{3}$ is rational, then $\sqrt{3}$ is rational.

This is FALSE since $\sim P$ and $Q$ are true and hence $((\sim P) \wedge Q)$ is true. However, $R$ is false; and so we have a true statement implying a false statement. The implication is hence false.

(d) $(P \vee Q) \Rightarrow (\sim R)$

In words: If $\sqrt{2}$ is rational or $\frac{2}{3}$ is rational, then $\sqrt{3}$ is not rational.

This is TRUE since $Q$ is true makes $P \vee Q$ true. Also, since $\sim R$ is true, we have a true statement implying a true statement. Hence, the implication is true.

46. For statements $P$ and $Q$, show that $P \Rightarrow (P \vee Q)$ is a tautology.

The following truth table shows that $P \Rightarrow (P \vee Q)$ is a tautology. This is indicated by the fact that the statement has true (T) values in every instance.

| $P$ | $Q$ | $P \vee Q$ | $P \Rightarrow (P \vee Q)$ |
|---|---|---|---|
| T | T | T | T |
| T | F | T | T |
| F | T | T | T |
| F | F | F | T |